

# Zero-Temperature Attenuation and Transverse Spin Dynamics in Fermi Liquids. II. Dilute Fermi Systems

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*This is the second in a series of papers on a consistent microscopic theory of transverse dynamics in spin-polarized or binary Fermi liquids. We demonstrate when and how the exact theory of Ref. 1 reduces to the conventional theory of highly polarized degenerate low-density Fermi liquids and gases. In the lowest approximations, i.e. for an ideal polarized Fermi gas and in the first (Born) order, our theory assumes the standard form. In the next order in density and/or interaction, the main equations still have a fairly conventional form, though they already contain the peculiar zero-temperature attenuation which is missing in the standard theory. This attenuation can be incorporated into the standard Fermi liquid formalism by adding an imaginary part to a mixed spin component of the Landau interaction function. The source of this imaginary contribution at  $T=0$  is a pole in the integral expression for the Landau interaction function (the situation is very similar to the case of collisionless Landau damping). In the next order, the standard theory fails completely, and even the form of the equations of transverse dynamics becomes very unconventional. We calculated explicitly the parameters of transverse spin dynamics and the spectrum of spin waves, including the zero-temperature attenuation, and, as a by-product, the polarization dependencies of thermodynamic parameters. The calculation includes a possible non-locality of the interaction. An application of the results to  $^3\text{He}$ - $^4\text{He}$  mixtures covers the non-locality in the direct interaction channel as well as the non-locality and retardation associated with a phonon-mediated part of particles' interaction.*

## 1. INTRODUCTION

In the first paper of this series<sup>1</sup> (referred below as I) we have developed a general microscopic theory of transverse dynamics in binary or spin polarized Fermi liquids.

It has been known for many years that a naive straightforward approach in the spirit of the Landau theory of Fermi liquids cannot and should not be used for a description of *transverse* (off-diagonal in spin) phenomena in spin-polarized Fermi liquids with high degrees of polarization. The reasons are associated with a strong spatial and temporal non-locality of the “exact” theory, influence of complex off-shell terms, and strong attenuation of all possible off-diagonal transverse spin states which correspond to transitions between noticeably shifted from each other Fermi surfaces for spin-ups and spin-downs. Unfortunately, as far as we know, all this information has never been published in a concise form, and has circulated mostly in the form of folklore (a very brief summary can be found, for example, in Ref. 2). As a result, one can see from time to time the attempts to describe the transverse spin dynamics in highly polarized Fermi liquids on the basis of the standard Landau theory. Such an approach has been very successful at low spin polarizations,<sup>3-5</sup> but becomes unjustifiable and fails completely when the polarization increases.

In I we have confirmed that it is impossible to get a Landau-like mean field equation for an off-diagonal spin component of single-particle (or quasi-particle) distribution,  $n_{\uparrow\downarrow}$ , from exact microscopic equations. Such a closed equation simply does not exist, and, therefore, cannot be derived with the help of any renormalization of relevant microscopic characteristics. The reason is a strong temporal non-locality, because of which the exact equation in the off-diagonal Green’s function  $G_{\uparrow\downarrow}(P)$  has not a single, but two distinctly separate peaks on two different energy surfaces (i.e. for two different values of the temporal component of the 4-momentum  $P$ ). Therefore, it is impossible to reduce the exact equation in  $G_{\uparrow\downarrow}(P)$  to just one equation in a single transverse density  $n_{\uparrow\downarrow}(\mathbf{p})$  which is a function of the 3-momentum  $\mathbf{p}$  and does not reflect any temporal non-locality.

However, the situation is not completely hopeless. We have shown that the exact equation in  $G_{\uparrow\downarrow}(P)$  can be reduced to a set of two coupled non-local equations in some partial transverse *pseudo*-densities. Because of strong intrinsic attenuation, these *pseudo*-densities do not represent any long-lived quasi-particles and are complex. The two transverse *pseudo*-densities,  $\delta n_{\uparrow}(\mathbf{p}) \equiv \delta n_{\uparrow\downarrow}^{(\uparrow)}(\mathbf{p})$  and  $\delta n_{\downarrow}(\mathbf{p}) \equiv \delta n_{\uparrow\downarrow}^{(\downarrow)}(\mathbf{p})$ , are, generically, the separate momentum distributions for transverse magnetic moments  $\delta n_{\uparrow\downarrow}(\mathbf{p}) = \text{Tr}_{\sigma} \hat{\sigma}^+ \delta \hat{n}_{\sigma}(\mathbf{p})$  originating from slightly tilted spins of spin-up and spin-down particles ( $\hat{n}_{\sigma}$  is the single-particle density matrix). Taken separately, these transverse *pseudo*-densities  $\delta n_{\uparrow}(\mathbf{p}) \equiv \delta n_{\uparrow\downarrow}^{(\uparrow)}(\mathbf{p})$  and  $\delta n_{\downarrow}(\mathbf{p}) \equiv \delta n_{\uparrow\downarrow}^{(\downarrow)}(\mathbf{p})$  do not reflect any observables: a corresponding observable is the momentum distribution of transverse magnetization which is a combination of these two *pseudo*-densities. For this reason, the equations of Ref. 1 cannot be considered as some macroscopic Landau-like equations; in

some sense, the fact that these equations resemble the doubled Landau equations is somewhat misleading.

The only assumption of our microscopic theory is that the *pure* spin states obey the conventional microscopic Landau theory, i.e. that the equilibrium Green's functions for the pure states,  $G_{\uparrow\uparrow}$  and  $G_{\downarrow\downarrow}$ , have singularities (poles) on their respective Fermi surfaces. [This means also that the description of longitudinal processes, not accompanied by changes in the direction of magnetization, keeps the standard Landau form for spin-up and spin-down quasi-particles.] This is quite a natural assumption, though, of course, it excludes different unconventional Fermi liquids.

An important consequence of our theory is a more clear understanding of the nature of a peculiar zero-temperature transverse attenuation in transverse spin dynamics. Of course, there is no real relaxation at zero temperatures. Probably, the best qualitative description of such a zero-temperature attenuation can be given as a dephasing of inhomogeneous precession between the above partial transverse magnetic moments (*pseudo*-densities). This dephasing is caused by a difference in precession frequencies (or effective molecular fields) for tilted spin-ups and spin-downs which results in an inhomogeneous broadening.

Another consequence of the doubling of transverse equations at high polarizations, as opposed to non-polarized (or slightly polarized) systems, is an increase in number of components of the transverse Fermi-liquid interaction function. What is more, the transverse spin dynamics is characterized not by one, but by a set of different transverse relaxation times. It turns out, that the transverse Fermi liquid interaction is described by a non-local  $2 \times 2$  operator. In contrast to the conventional Landau theory, the components of this interaction operator are complex, and are not given as some limit of the full vertex, but are related to the exact irreducible vertex function via some complicated integral equation.

Though Ref. I contains a consistent general theory of transverse spin dynamics in spin-polarized Fermi liquids, the corresponding equations are very cumbersome and not very transparent. Therefore, it is very important to understand how this theory reduces to a standard Landau theory in different limiting cases. It is also highly desirable to illustrate the general results<sup>1</sup> by some examples for which it would be possible to get the final equations in a compact and more transparent form easily accessible for experimental applications. Since one does not have exact general expressions for mass operators or vertex functions in dense Fermi liquids, one should turn either to model systems, or to Fermi systems with some inherent expansion parameters. Probably, the best choice is given by a study of low density Fermi systems.

In case of low density Fermi liquids—basically, degenerate non-ideal

Fermi gases—one has a natural expansion parameter, namely, the density of fermions. What is even more important, the expansion in fermion density  $N$  (more exactly,  $N^{1/3}$ ) formally coincides with the perturbation expansion making a perturbative approach applicable even for strongly interacting dilute systems (see, for example, the review<sup>6</sup>). The dimensionless expansion parameter is  $aN^{1/3} \sim ap_F/\hbar$  ( $a$  is the bare  $s$ -wave scattering length,  $p_F$  is the Fermi momentum), and one can consider a density expansion as if it is an expansion in  $a$ . The first two density corrections are proportional to  $a$  and  $a^2$ . However, the next term should contain not only  $a^3$ , but also a contribution proportional to the  $p$ -wave scattering constant,  $b$ . After two more density expansion terms, one should include the  $d$ -wave processes, and so on.

As a result, one gets a density expansion which is constructed in the form of a perturbative series where higher order terms should include scattering lengths in channels with higher and higher angular numbers. The corresponding perturbative procedure is quite transparent,<sup>7,8</sup> and fails only if the interaction at large distances does not vanish fast enough to justify an angular harmonics/momentum expansion in higher orders.

For most of the practical purposes, it is sufficient to truncate such a density expansion for not too dense systems after the second order interaction terms (i.e. to retain the terms which contain the parameter  $a$  and  $a^2$  exclusively). Generally, such an accuracy is sufficient to reproduce nearly the whole variety of phenomena in Fermi liquids. However, such an approach is not just a theoretical exercise which provides one with a reasonable model for an illustration of some phenomena in Fermi liquids: this approach gives an adequate description of observable effects in a gas of  $^3\text{He}$  quasi-particles in not very concentrated liquid  $^3\text{He}\uparrow$ - $^4\text{He}$  mixtures.<sup>6,9</sup>

In this paper we analyze our general equations<sup>1</sup> in case of low density Fermi liquids and gases with an emphasis on transverse spin dynamics and, especially, zero-temperature attenuation in spin-polarized  $^3\text{He}\uparrow$ -HeII. The fact that the *transverse* relaxation time and *transverse* spin diffusion coefficient remain finite at  $T \rightarrow 0$ , in contrast to the corresponding *longitudinal* parameters which increase as  $1/T^2$ , was predicted by one of the authors about a decade ago (see review<sup>6</sup> and references therein). Later the existence of this zero-temperature transverse attenuation was confirmed by variational calculations for dilute systems in the Born approximation.<sup>10,11</sup> We will get the values of the zero-temperature attenuation and transverse diffusion coefficient from the exact equations<sup>1</sup> in the low-density limit.

In the next Section we will show how the equations<sup>1</sup> reduce to a standard theory of spin-polarized gases in the limit of very low densities. In Sec. 3 we will calculate all the relevant Fermi-liquid parameters, including the zero-temperature attenuation and the spectrum of spin waves, up to the

second-order perturbation/density terms. As a by-product, we will get the polarization dependencies of thermodynamic parameters. With this accuracy, the zero-temperature attenuation can still be incorporated into the Fermi liquid description. We will point out the terms which are responsible for a deviation from the "standard" picture in the next order. Sec. 4 deals with an influence of spatial non-locality on a zero-temperature attenuation. The non-locality can be caused by a long-range interaction or by retardation effects which are associated with a phonon-mediated part of  ${}^3\text{He}$ - ${}^3\text{He}$  interaction in  ${}^3\text{He}\uparrow$ - ${}^4\text{He}$  liquid mixtures. The last Section contains a brief summary and a discussion of the results for  ${}^3\text{He}\uparrow$ - ${}^4\text{He}$  mixtures. Details of computational results are given in the Appendices.

## 2. GENERAL THEORY IN THE LOW DENSITY LIMIT: A LINK WITH STANDARD EQUATIONS

Of course, for very dilute gases the dynamics and transport equations can and should be written via a usual single-particle density matrix, and not some *pseudo*-densities, regardless of whether the gas is spin-polarized or not. This means that in this case our general theory should somehow reduce to a fairly standard kinetic theory for a degenerate Fermi gas. In this Section we will demonstrate how the general equations of Ref. 1 reduce to the standard Landau-like theory in the limit of very low densities.

First, let us give the main equations of I and all relevant definitions. In I we started from the exact Dyson equation in the mixed spin component of the single-particle Green's function  $G_{\uparrow\downarrow}(P)$  at  $T=0$  and managed to reduce this equation to the following set of two coupled equations in partial transverse *pseudo*-densities  $\delta n_{\uparrow}$  and  $\delta n_{\downarrow}$ , Eqs. (I.34):

$$\begin{aligned}
 W_{\downarrow}(K; \mathbf{p}) \delta n_{\downarrow}(\mathbf{p}) &= \frac{1}{2} \int [F_{\downarrow\downarrow}(K; \mathbf{p}, \mathbf{p}') \delta n_{\downarrow}(\mathbf{p}') \\
 &\quad + F_{\downarrow\uparrow}(K; \mathbf{p}, \mathbf{p}') \delta n_{\uparrow}(\mathbf{p}')] (\theta_{\downarrow}(\mathbf{p}') - \theta_{\uparrow}(\mathbf{p}')) \frac{d^3 p'}{(2\pi)^3} \\
 W_{\uparrow}(K; \mathbf{p}) \delta n_{\uparrow}(\mathbf{p}) &= \frac{1}{2} \int [F_{\uparrow\downarrow}(K; \mathbf{p}, \mathbf{p}') \delta n_{\downarrow}(\mathbf{p}') \\
 &\quad + F_{\uparrow\uparrow}(K; \mathbf{p}, \mathbf{p}') \delta n_{\uparrow}(\mathbf{p}')] (\theta_{\downarrow}(\mathbf{p}') - \theta_{\uparrow}(\mathbf{p}')) \frac{d^3 p'}{(2\pi)^3}
 \end{aligned} \tag{1}$$

Here the eigenvectors are the transverse *pseudo*-densities,  $\delta n_{\uparrow}(\mathbf{p})$  and  $\delta n_{\downarrow}(\mathbf{p})$ , which are defined via the off-diagonal (mixed) spin component of the single-particle Green's function as

$$\delta n_{\uparrow}(\mathbf{p}) = g(P_{\uparrow}), \quad \delta n_{\downarrow}(\mathbf{p}) = g(P_{\downarrow})$$

$$\delta G_{\uparrow\downarrow}(P; t, \mathbf{r}) = g(P) \left[ G_{\downarrow\downarrow} \left( P + \frac{K}{2} \right) - G_{\uparrow\uparrow} \left( P - \frac{K}{2} \right) \right] \exp(i\mathbf{k}\mathbf{r} - i\omega t) \quad (2)$$

where  $P = (p_0, \mathbf{p})$  is the 4-momentum, the wave 4-vector  $K = (\omega, \mathbf{k})$  introduces temporal and spatial non-localities in the theory, and the energy components of the 4-vectors  $P_{\uparrow}$  and  $P_{\downarrow}$  are determined by the single-particle energy spectra  $\varepsilon_{\uparrow}$  and  $\varepsilon_{\downarrow}$ :

$$K = (\omega, \mathbf{k}), \quad P_{\downarrow} = \left( \varepsilon_{\downarrow} \left( \mathbf{p} + \frac{\mathbf{k}}{2} \right) - \mu - \frac{\delta\omega}{2} - \beta H, \mathbf{p} \right)$$

$$P_{\uparrow} = \left( \varepsilon_{\uparrow} \left( \mathbf{p} - \frac{\mathbf{k}}{2} \right) - \mu + \frac{\delta\omega}{2} + \beta H, \mathbf{p} \right), \quad \delta\omega = \omega - 2\beta H \quad (3)$$

$$\theta_{\downarrow}(\mathbf{p}) = \theta \left( \varepsilon_{\downarrow} \left( \mathbf{p} + \frac{\mathbf{k}}{2} \right) - \mu \right), \quad \theta_{\uparrow}(\mathbf{p}) = \theta \left( \varepsilon_{\uparrow} \left( \mathbf{p} - \frac{\mathbf{k}}{2} \right) - \mu \right)$$

The generalized Liouville operators, i.e. the functions  $W_{\uparrow,\downarrow}$  in the left hand side of Eqs. (1), are expressed through the single-particle energies,  $\varepsilon_{\uparrow}$  and  $\varepsilon_{\downarrow}$ , and some *pseudo-energies*,  $\tilde{\varepsilon}_{\uparrow}$  and  $\tilde{\varepsilon}_{\downarrow}$  (here and below  $\hbar = 1$ ):

$$W_{\downarrow}(K; \mathbf{p}) = \left[ \delta\omega + 2\beta H - \varepsilon_{\downarrow} \left( \mathbf{p} + \frac{\mathbf{k}}{2} \right) + \tilde{\varepsilon}_{\uparrow} \left( \mathbf{p} - \frac{\mathbf{k}}{2} \right) \right] Z_{\downarrow} \left( \mathbf{p} + \frac{\mathbf{k}}{2} \right)$$

$$W_{\uparrow}(K; \mathbf{p}) = \left[ \delta\omega + 2\beta H - \tilde{\varepsilon}_{\downarrow} \left( \mathbf{p} + \frac{\mathbf{k}}{2} \right) + \varepsilon_{\uparrow} \left( \mathbf{p} - \frac{\mathbf{k}}{2} \right) \right] Z_{\uparrow} \left( \mathbf{p} - \frac{\mathbf{k}}{2} \right) \quad (4)$$

while the energies and *pseudo-energies* themselves are defined with the help of mass operators for *pure* spin-up and spin-down states of particles  $\Sigma_{\uparrow\uparrow}(P)$  and  $\Sigma_{\downarrow\downarrow}(P)$  as

$$\varepsilon_{\uparrow}(\mathbf{p}) = \frac{p^2}{2m} + \Sigma_{\uparrow\uparrow}(\varepsilon_{\uparrow}(\mathbf{p}) - \mu, \mathbf{p}) - \beta H$$

$$\tilde{\varepsilon}_{\uparrow}(\mathbf{p}) = \frac{p^2}{2m} + \Sigma_{\uparrow\uparrow}(\varepsilon_{\downarrow}(\mathbf{p} + \mathbf{k}) - \mu - \delta\omega - 2\beta H, \mathbf{p}) - \beta H$$

$$\varepsilon_{\downarrow}(\mathbf{p}) = \frac{p^2}{2m} + \Sigma_{\downarrow\downarrow}(\varepsilon_{\downarrow}(\mathbf{p}) - \mu, \mathbf{p}) + \beta H$$

$$\tilde{\varepsilon}_{\downarrow}(\mathbf{p}) = \frac{p^2}{2m} + \Sigma_{\downarrow\downarrow}(\varepsilon_{\uparrow}(\mathbf{p} - \mathbf{k}) - \mu + \delta\omega + 2\beta H, \mathbf{p}) + \beta H \quad (5)$$

and the renormalization coefficients  $Z_{\uparrow,\downarrow}$  can be expressed according to Eq. (I.29) as

$$1 - \frac{1}{Z_{\uparrow\downarrow}(\mathbf{p})} = \frac{\partial \Sigma_{\uparrow\downarrow\downarrow}(p_0 = \varepsilon_{\uparrow\downarrow}(\mathbf{p}), \mathbf{p})}{\partial p_0} \quad (6)$$

The 4-component interaction function in the r.h.s. of Eq. (1) serves as a generalized Landau function for transverse phenomena, and is equal to (I.36)

$$\begin{aligned} F_{\downarrow\downarrow}(K; \mathbf{p}, \mathbf{p}') &= Z_{\downarrow}\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) Z_{\downarrow}\left(\mathbf{p}' + \frac{\mathbf{k}}{2}\right) \mathfrak{C}(K; P_{\downarrow}, P'_{\downarrow}) \\ F_{\downarrow\uparrow}(K; \mathbf{p}, \mathbf{p}') &= Z_{\downarrow}\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) Z_{\uparrow}\left(\mathbf{p}' - \frac{\mathbf{k}}{2}\right) \mathfrak{C}(K; P_{\downarrow}, P'_{\uparrow}) \\ F_{\uparrow\downarrow}(K; \mathbf{p}, \mathbf{p}') &= Z_{\uparrow}\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) Z_{\downarrow}\left(\mathbf{p}' + \frac{\mathbf{k}}{2}\right) \mathfrak{C}(K; P_{\uparrow}, P'_{\downarrow}) \\ F_{\uparrow\uparrow}(K; \mathbf{p}, \mathbf{p}') &= Z_{\uparrow}\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) Z_{\uparrow}\left(\mathbf{p}' - \frac{\mathbf{k}}{2}\right) \mathfrak{C}(K; P_{\uparrow}, P'_{\uparrow}) \end{aligned} \quad (7)$$

where the generating function  $\mathfrak{C}(K; P, P')$  is related to the mixed spin component of the *irreducible* vertex function  $\tilde{\Gamma}_{\uparrow\downarrow\downarrow\uparrow}(K; P, P')$  via the integral equation (I.23)

$$\begin{aligned} \mathfrak{C}(K; P, P') &= \tilde{\Gamma}_{\uparrow\downarrow\downarrow\uparrow}(K; P, P') + \int \tilde{\Gamma}_{\uparrow\downarrow\downarrow\uparrow}(K; P, Q) \phi_r(Q + K/2, Q - K/2) \\ &\quad \times \mathfrak{C}(K; Q, P') \frac{d^4 Q}{(2\pi)^4} \end{aligned} \quad (8)$$

and  $\phi_r$  is the regular part of the Green's functions defined as

$$\begin{aligned} G_{\downarrow\downarrow}\left(P + \frac{K}{2}\right) - G_{\uparrow\uparrow}\left(P - \frac{K}{2}\right) &= i\phi_r\left(P + \frac{K}{2}, P - \frac{K}{2}\right) \left[ \delta\omega - \frac{\mathbf{p}\mathbf{k}}{m} - \Sigma_{\downarrow\downarrow}\left(P + \frac{K}{2}\right) + \Sigma_{\uparrow\uparrow}\left(P - \frac{K}{2}\right) \right] \\ &\quad + i\phi_s\left(P + \frac{K}{2}, P - \frac{K}{2}\right) \\ \phi_s\left(P + \frac{K}{2}, P - \frac{K}{2}\right) &= \pi \left[ \theta\left(\varepsilon_{\downarrow}\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) - \mu\right) - \theta\left(\varepsilon_{\uparrow}\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) - \mu\right) \right] \\ &\quad \times \left[ Z_{\downarrow}\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) \delta\left(p_0 + \frac{\delta\omega}{2} + \beta H - \varepsilon_{\downarrow}\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) + \mu\right) \right. \\ &\quad \left. + Z_{\uparrow}\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) \delta\left(p_0 - \frac{\delta\omega}{2} - \beta H - \varepsilon_{\uparrow}\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) + \mu\right) \right] \end{aligned} \quad (9)$$

Before turning to low density expansions, it is essential to analyze in more detail the difference between *pseudo*-energies and energies (5). One can rewrite Eqs. (5) in a slightly different form:

$$\begin{aligned}\tilde{\varepsilon}_\uparrow\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) &= \frac{(\mathbf{p}-\mathbf{k}/2)^2}{2m} + \Sigma_{\uparrow\uparrow}\left(\varepsilon_\uparrow\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) - \mu - \delta\omega\right. \\ &\quad \left.+ 2(\beta_1(\mathbf{p}; \mathbf{k}, H) - \beta)H, \mathbf{p}-\frac{\mathbf{k}}{2}\right) - \beta H \\ \tilde{\varepsilon}_\downarrow\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right) &= \frac{(\mathbf{p}+\mathbf{k}/2)^2}{2m} + \Sigma_{\downarrow\downarrow}\left(\varepsilon_\downarrow\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right) - \mu + \delta\omega\right. \\ &\quad \left.- 2(\beta_1(\mathbf{p}; \mathbf{k}, H) - \beta)H, \mathbf{p}+\frac{\mathbf{k}}{2}\right) + \beta H\end{aligned}\quad (10)$$

where the function  $\beta_1$  shows the energy difference between spin-up and spin-down states (including the interaction), and is, in some sense, the dressed magnetic moment,

$$\beta_1(\mathbf{p}; \mathbf{k}, H) = \frac{1}{2H} \left[ \varepsilon_\uparrow\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) - \varepsilon_\downarrow\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right) \right] \quad (11)$$

This function characterizes the magnetic susceptibility of the Fermi liquid. If the particle interaction and/or density are weak, then  $\varepsilon_\uparrow - \varepsilon_\downarrow \rightarrow 2\beta H$ , and  $\beta_1(\mathbf{p}; \mathbf{k}, H)$  is close to the bare magnetic moment  $\beta$ . The comparison of Eqs. (5) and (10) shows that the difference between *pseudo*-energies and energies disappears only when  $(\beta_1 - \beta)H$  is negligible. This can be true either when the polarization is low (small  $H$ ), or the density and/or interaction are small and  $\beta_1$  is close to its bare value  $\beta$ . In either of these cases, the expansion of Eqs. (10) at small  $\mathbf{k}$  and  $\delta\omega$  looks like

$$\begin{aligned}\tilde{\varepsilon}_\uparrow\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) &\approx \varepsilon_\uparrow\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) + [2(\beta_1(\mathbf{p}; \mathbf{k}, H) - \beta)H - \delta\omega] \left[ 1 - Z_\uparrow^{-1}\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) \right] \\ \tilde{\varepsilon}_\downarrow\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right) &\approx \varepsilon_\downarrow\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right) - [2(\beta_1(\mathbf{p}; \mathbf{k}, H) - \beta)H - \delta\omega] \left[ 1 - Z_\downarrow^{-1}\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right) \right]\end{aligned}\quad (12)$$

Then the non-local Liouville operators  $W_{\uparrow,\downarrow}$  (4) become equal to

$$\begin{aligned}W_\downarrow(K; \mathbf{p}) &\approx \left[ \delta\omega + 2(\beta - \beta_1(\mathbf{p}; 0, H))H - \frac{\mathbf{k}\mathbf{v}_\downarrow + \mathbf{k}\mathbf{v}_\uparrow}{2} \right] \\ &\quad \times Z_\uparrow^{-1}\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) Z_\downarrow\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right)\end{aligned}$$

$$W_{\uparrow}(K; \mathbf{p}) \approx \left[ \delta\omega + 2(\beta - \beta_1(\mathbf{p}; 0, H))H - \frac{\mathbf{k}\mathbf{v}_{\downarrow} + \mathbf{k}\mathbf{v}_{\uparrow}}{2} \right] \\ \times Z_{\downarrow}^{-1} \left( \mathbf{p} + \frac{\mathbf{k}}{2} \right) Z_{\uparrow} \left( \mathbf{p} - \frac{\mathbf{k}}{2} \right) \quad (13)$$

and are very close to each other. Therefore, the main key to the collapse of two equations (1) into a single standard Fermi liquid equation is the smallness of the parameter  $(\beta_1 - \beta)H$ . This parameter is small only in cases of low polarization or low density/interaction when  $\beta_1$  is close to  $\beta$ .

Now let us demonstrate why and how the above equations reduce to a single standard equation of the Landau Fermi liquid theory of spin-polarized degenerate Fermi gases<sup>6</sup> in case of very low densities. More specifically, we will analyze how the set of equations (1) reduces to a single equation in a transverse density of quasi-particles (magnetic moment distribution). We will do it by a density expansion, which is formally equivalent to an expansion in the scattering length  $a$ , up to the second order terms proportional to  $a^2$ . We will denote different order terms in the corresponding equations by upper indices: the upper index <sup>(0)</sup> will mean the zeroth-order term corresponding to an ideal non-interacting Fermi gas, <sup>(1)</sup> will correspond to the first-order density/interaction correction linear in  $aN^{1/3}$ , <sup>(2)</sup> will stand for  $(aN^{1/3})^2$  contributions, and so on.

As we will see below (Appendix B), the renormalization functions for pure spin states  $Z_{\uparrow, \downarrow}$  (6) differ from 1 only in the second order in density:  $Z_{\uparrow, \downarrow}^{(0)} = 1$ ,  $Z_{\uparrow, \downarrow}^{(1)} = 0$ ,  $Z_{\uparrow, \downarrow}^{(2)} \propto a^2 \neq 0$ . Since  $\beta_1 - \beta$  is also small in density, this means that the functions  $W_{\uparrow, \downarrow}$  (13) are equal to each other up to the terms of the order  $a^2$ :  $W_{\uparrow}^{(0)} = W_{\downarrow}^{(0)}$ ,  $W_{\uparrow}^{(1)} = W_{\downarrow}^{(1)}$ ,  $W_{\uparrow}^{(2)} = W_{\downarrow}^{(2)}$  (the exact expressions will be given below). Therefore, the left hand sides of Eqs. (1) are the same up to the second-order density terms.

On the other hand, the density expansion of the irreducible vertex function  $\tilde{\Gamma}_{\uparrow\downarrow, \uparrow\uparrow}(K; P, P')$  starts from the linear in interaction term,  $\tilde{\Gamma}^{(1)}$ , while the regular part  $\phi_r$  of the Green's functions  $\phi_r^{(0)} = \phi_r^{(1)} = 0$ . Therefore, according to Eq. (8),

$$\mathfrak{G}^{(0)}(K; P, P') = 0, \quad \mathfrak{G}^{(1)}(K; P, P') = \tilde{\Gamma}_{\uparrow\downarrow, \uparrow\uparrow}^{(1)}(K; P, P'), \\ \mathfrak{G}^{(2)}(K; P, P') = \tilde{\Gamma}_{\uparrow\downarrow, \uparrow\uparrow}^{(2)}(K; P, P') \quad (14)$$

What is more, we will see below that the first order term in the vertex part,  $\tilde{\Gamma}^{(1)}$ , does not depend on momenta,  $\tilde{\Gamma}^{(1)}(K; P, P') = \text{const}$ . As a result, the difference between components of the Fermi liquid operator  $F_{ik}$  (7) can be caused, up to the second order in density, only by some zeroth-order difference in arguments  $P_{\uparrow}$  and  $P_{\downarrow}$  for  $\tilde{\Gamma}^{(2)}(K; P, P')$  (14). However, in the

zeroth order,  $\varepsilon_{\uparrow}^{(0)} + \beta H = \varepsilon_{\downarrow}^{(0)} - \beta H = p^2/2m$ , and, with the same accuracy,  $P_{\uparrow}$  and  $P_{\downarrow}$  in the arguments of  $\tilde{\Gamma}^{(2)}(K; P, P')$  should be considered equal,  $P_{\uparrow}^{(0)} - K/2 = P_{\downarrow}^{(0)} + K/2$ . As a result,

$$\begin{aligned} F_{\uparrow\uparrow}^{(0)}(K; \mathbf{p}, \mathbf{p}') &= F_{\downarrow\downarrow}^{(0)}(K; \mathbf{p}, \mathbf{p}') = F_{\downarrow\uparrow}^{(0)}(K; \mathbf{p}, \mathbf{p}') = F_{\uparrow\downarrow}^{(0)}(K; \mathbf{p}, \mathbf{p}') = 0, \\ F_{\uparrow\uparrow}^{(1)}(K; \mathbf{p}, \mathbf{p}') &= F_{\downarrow\downarrow}^{(1)}(K; \mathbf{p}, \mathbf{p}') = F_{\downarrow\uparrow}^{(1)}(K; \mathbf{p}, \mathbf{p}') = F_{\uparrow\downarrow}^{(1)}(K; \mathbf{p}, \mathbf{p}') \\ &\equiv \tilde{\Gamma}_{\uparrow\downarrow, \downarrow\uparrow}^{(1)}(K; P, P') = \text{const}, \\ F_{\uparrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') &= F_{\downarrow\downarrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') = F_{\downarrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') = F_{\uparrow\downarrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') \\ &= \tilde{\Gamma}_{\uparrow\downarrow, \downarrow\uparrow}^{(2)}(K; P, P') \end{aligned} \quad (15)$$

Therefore, up to the second order in density, the set of equations (1) reduces to a single kinetic equation in transverse density  $n_{\uparrow\downarrow}(\mathbf{p}) = n_{\downarrow}(\mathbf{p}) + n_{\uparrow}(\mathbf{p})$ :

$$W(K; \mathbf{p}) \delta n_{\uparrow\downarrow}(\mathbf{p}) = \int \tilde{\Gamma}_{\uparrow\downarrow, \downarrow\uparrow}(K; P, P') \delta n_{\uparrow\downarrow}(\mathbf{p}') (\theta_{\downarrow}(\mathbf{p}') - \theta_{\uparrow}(\mathbf{p}')) \frac{d^3 p'}{(2\pi)^3} \quad (16)$$

The explicit expressions for the functions  $W$  and  $\tilde{\Gamma}_{\uparrow\downarrow, \downarrow\uparrow}$  in Eq. (16) will be given in the next Section. This equation plays the role of the transport equation for the transverse spin density in a very low density limit (i.e. in the gas limit).

Generally, the doubling of equations is associated with an up-down asymmetry (somewhat similar to a particle-hole asymmetry) for dressed particles, and appears only starting from the third order in density. In the third order

$$\begin{aligned} F_{\uparrow\uparrow}^{(3)}(K; \mathbf{p}, \mathbf{p}') &= \mathfrak{C}^{(3)}(K; P_{\uparrow}^{(0)}, P_{\uparrow}^{(0)}) \\ &+ \left[ Z_{\uparrow}^{(2)} \left( \mathbf{p} - \frac{\mathbf{k}}{2} \right) + Z_{\uparrow}^{(2)} \left( \mathbf{p}' - \frac{\mathbf{k}}{2} \right) \right] \tilde{\Gamma}_{\uparrow\downarrow, \downarrow\uparrow}^{(1)}(K; P, P') \\ &+ \tilde{\Gamma}_{\uparrow\downarrow, \downarrow\uparrow}^{(2)}(K; P_{\uparrow}^{(1)}, P_{\uparrow}^{(1)}) \end{aligned} \quad (17)$$

and similarly for other three components of the matrix  $\hat{F}^{(3)}$ . One can also write the same type of equations for  $W_{\uparrow, \downarrow}$ . Only the last term in Eq. (17) is different for different components of  $F_{ik}$ . It is obvious that with this accuracy Eqs. (1) do not merge anymore (see Eqs. (25), (26) below).

One should keep in mind that the calculation of the first term in the r.h.s. of Eq. (17) is not very simple: it should involve not only the standard third order  $s$ -wave scattering term, but also the first order  $p$ -wave term *and* the retardation to the first order  $s$ -wave term.<sup>9</sup>

Note, that the zero temperature attenuation shows up for the first time in the second order while the set of Eqs. (1) is still degenerate and reduces to the single Eq. (16) (see the next Section).

### 3. SPIN-POLARIZED LOW DENSITY FERMI LIQUIDS WITH SHORT-RANGE INTERACTION AT $T = 0$

In this section we will calculate explicitly all the relevant parameters of transverse spin dynamics as an expansion in density up to the second order.

The main order in density for dilute systems corresponds to an ideal non-interacting polarized Fermi gas, and is trivially described by the Stoner equations (see, for example, the reviews<sup>6,9</sup>).

The first order density/interaction terms are also quite simple since with this accuracy the scattering amplitude— $a$  is a constant which does not depend on momenta of interacting particles,<sup>7</sup> and the corresponding (irreducible) vertex is a constant,<sup>6,9</sup>  $\tilde{\Gamma}^{(1)} = -4\pi a/m$ . Therefore,

$$\begin{aligned} \Sigma_{\uparrow\uparrow}^{(0)} = \Sigma_{\downarrow\downarrow}^{(0)} = \tilde{\Gamma}_{\uparrow\downarrow,\downarrow\uparrow}^{(0)} = 0, \quad \varepsilon_{\uparrow,\downarrow}^{(0)}(\mathbf{p}) = \tilde{\varepsilon}_{\uparrow,\downarrow}^{(0)}(\mathbf{p}) = \frac{p^2}{2m} \mp \beta H \\ \Sigma_{\uparrow\uparrow}^{(1)}(p_0, \mathbf{p}) = \frac{4\pi a}{m} N_{\downarrow}, \quad \Sigma_{\downarrow\downarrow}^{(1)}(p_0, \mathbf{p}) = \frac{4\pi a}{m} N_{\uparrow}, \quad \tilde{\Gamma}_{\uparrow\downarrow,\downarrow\uparrow}^{(1)}(K; P, P') = -\frac{4\pi a}{m}, \\ \varepsilon_{\uparrow,\downarrow}^{(1)}(\mathbf{p}) = \tilde{\varepsilon}_{\uparrow,\downarrow}^{(1)}(\mathbf{p}) = \frac{4\pi a}{m} N_{\downarrow,\uparrow}, \quad \mathbf{v}_{\uparrow}^{(0)} = \mathbf{v}_{\downarrow}^{(0)} = \frac{\mathbf{p}}{m}, \quad \mathbf{v}_{\uparrow}^{(1)} = \mathbf{v}_{\downarrow}^{(1)} = 0 \end{aligned} \quad (18)$$

where  $N_{\uparrow}$  and  $N_{\downarrow}$  are the numbers of spin-up and spin-down particles per unit volume. In this approximation, the set of equations (1) reduces, as it was discussed in the previous Section, to a single equation (16), which, in turn, obtains the standard form<sup>6</sup>

$$\left( \delta\omega - \frac{\mathbf{p}\mathbf{k}}{m} - \frac{4\pi a}{m} (N_{\uparrow} - N_{\downarrow}) \right) \delta n_{\uparrow,\downarrow}(\mathbf{p}) = \frac{4\pi a}{m} [\theta_{\uparrow}(\mathbf{p}) - \theta_{\downarrow}(\mathbf{p})] \int \delta n_{\uparrow,\downarrow}(\mathbf{p}') \frac{d^3 p'}{(2\pi)^3} \quad (19)$$

with the eigenvalue<sup>6,9</sup>

$$\delta\omega = \alpha^{(1)} k^2, \quad \alpha^{(1)} = \frac{(6\pi^2)^{2/3} N_{\uparrow}^{5/3} - N_{\downarrow}^{5/3}}{20\pi a m (N_{\uparrow} - N_{\downarrow})^2} \quad (20)$$

Quite naturally, there is no zero temperature attenuation within the first order: this approximation corresponds to a uniform momentum- and energy-independent molecular field. Therefore, the molecular field for tilted spin-ups and spin-downs is the same, and does not cause any dephasing or inhomogeneous broadening. [Note also, that since the attenuation, i.e. the imaginary part of the spectrum, should have a definite sign; it should start

from an even power of the interaction and, therefore, from the even power of the scattering length  $a$ .]

The calculations in the next order in density are much more cumbersome. Detailed calculations of the irreducible vertex and the mass operators in the second order in density are given in Appendices A and B. According to Eqs. (15), the generalized transverse Landau operators  $\hat{F}_{ik}^{(2)}$  are still equal to each other within this approximation, and are determined by the irreducible vertex (21). The single-particle energies and *pseudo*-energies, which enter the generalized Liouville operators (4), are easily calculated with the help of the self-energies  $\Sigma^{(2)}$  from Appendix B:

$$\begin{aligned}
 \varepsilon_{\uparrow}^{(2)}\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) &= \Sigma_{\uparrow\uparrow}^{(2)}\left(\frac{1}{2m}\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right)^2 - \beta H - \mu, \mathbf{p}-\frac{\mathbf{k}}{2}\right) \\
 \varepsilon_{\downarrow}^{(2)}\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right) &= \Sigma_{\downarrow\downarrow}^{(2)}\left(\frac{1}{2m}\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right)^2 + \beta H - \mu, \mathbf{p}+\frac{\mathbf{k}}{2}\right) \\
 \tilde{\varepsilon}_{\uparrow}^{(2)}\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) &= \Sigma_{\uparrow\uparrow}^{(2)}\left(\frac{1}{2m}\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right)^2 - \beta H - \mu - \delta\omega, \mathbf{p}-\frac{\mathbf{k}}{2}\right) \\
 \tilde{\varepsilon}_{\downarrow}^{(2)}\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right) &= \Sigma_{\downarrow\downarrow}^{(2)}\left(\frac{1}{2m}\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right)^2 + \beta H - \mu + \delta\omega, \mathbf{p}+\frac{\mathbf{k}}{2}\right)
 \end{aligned} \tag{21}$$

Results of calculations of the irreducible vertex, mass operators and the single-particle spectra (21) are presented in Appendices A, B and Figs. 6–8 in Appendix B.

As one can see from Eqs. (21), the difference between *pseudo*-energies  $\tilde{\varepsilon}$  and energies  $\varepsilon$  in this order in density disappears when  $\mathbf{k} = 0$  (note, that  $\delta\omega \propto k^2$ ). Therefore, the *pseudo*-energies are equal, up to the  $k^2$ -terms, to

$$\begin{aligned}
 \tilde{\varepsilon}_{\uparrow}^{(2)}\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) &= \varepsilon_{\uparrow}^{(2)}\left(\mathbf{p}-\frac{\mathbf{k}}{2}\right) - \left(\delta\omega - 2\frac{\mathbf{p}\mathbf{k}}{m}\right) Z_{\uparrow}^{(2)}(\mathbf{p}) \\
 &\quad + 2\frac{(\mathbf{p}\mathbf{k})^2}{m} \frac{\partial^2 \Sigma_{\uparrow\uparrow}^{(2)}(\varepsilon_{\uparrow}^{(0)}(\mathbf{p}) - \mu, \mathbf{p})}{\partial \varepsilon_{\uparrow}^{(0)2}} \\
 \tilde{\varepsilon}_{\downarrow}^{(2)}\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right) &= \varepsilon_{\downarrow}^{(2)}\left(\mathbf{p}+\frac{\mathbf{k}}{2}\right) + \left(\delta\omega - 2\frac{\mathbf{p}\mathbf{k}}{m}\right) Z_{\downarrow}^{(2)}(\mathbf{p}) \\
 &\quad + 2\frac{(\mathbf{p}\mathbf{k})^2}{m} \frac{\partial^2 \Sigma_{\downarrow\downarrow}^{(2)}(\varepsilon_{\downarrow}^{(0)}(\mathbf{p}) - \mu, \mathbf{p})}{\partial \varepsilon_{\downarrow}^{(0)2}}
 \end{aligned} \tag{22}$$

The renormalization functions  $Z_{\uparrow,\downarrow}^{(2)}$  (i.e. the residues of the Green's functions for pure spin-up and spin-down states, Eq. (6)) are given in Figures 8a,b (see Appendix B).

As a result, the Liouville operators  $W$  (4) obtain the form

$$\begin{aligned}
 W_{\uparrow}^{(0)}(K; \mathbf{p}) &= W_{\downarrow}^{(0)}(K; \mathbf{p}) = -\frac{\mathbf{p}\mathbf{k}}{m} \\
 W_{\uparrow}^{(1)}(K; \mathbf{p}) &= W_{\downarrow}^{(1)}(K; \mathbf{p}) = \alpha^{(1)}k^2 + \frac{4\pi a}{m}(N_{\downarrow} - N_{\uparrow}) \\
 W_{\uparrow, \downarrow}^{(2)}(K; \mathbf{p}) &= \varepsilon_{\uparrow}^{(2)}\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) - \varepsilon_{\downarrow}^{(2)}\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right) \\
 &\quad + \alpha^{(2)}k^2 \mp 2\frac{\mathbf{p}\mathbf{k}}{m}\left(Z_{\uparrow}^{(2)}\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) - Z_{\downarrow}^{(2)}\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right)\right) \\
 &\quad - \delta\omega Z_{\downarrow, \uparrow}^{(2)}\left(\mathbf{p} \pm \frac{\mathbf{k}}{2}\right) + 2\frac{(\mathbf{p}\mathbf{k})^2}{m}\frac{\partial^2 \Sigma_{\downarrow, \uparrow}^{(2)}(\varepsilon_{\downarrow, \uparrow}^{(0)}(\mathbf{p}) - \mu, \mathbf{p})}{\partial \varepsilon_{\downarrow, \uparrow}^{(0)2}}
 \end{aligned} \tag{23}$$

Though all the terms in the r.h.s. of the last of Eqs. (23) are formally of the same order of magnitude in  $a$ , the last three terms are of the higher orders in  $\mathbf{k}$ . It is quite obvious, that, eventually, the terms of the order  $a^2k$  will contribute to the spin-wave spectrum in the same order as  $a^3$ -terms without  $k$ , while the contribution from the terms  $a^2k^2$  is of the same order as from the fourth order terms  $a^4$ . Therefore, if we are interested only in two first terms in the spin-wave spectrum, Eq. (23) should be rewritten as

$$W_{\uparrow}^{(2)}(K; \mathbf{p}) = W_{\downarrow}^{(2)}(K; \mathbf{p}) = \varepsilon_{\uparrow}^{(2)}(\mathbf{p}) - \varepsilon_{\downarrow}^{(2)}(\mathbf{p}) + \alpha^{(2)}k^2 \tag{24}$$

The terms

$$W_{\uparrow, \downarrow}^{(3)}(K; \mathbf{p}) = \mp 2\frac{\mathbf{p}\mathbf{k}}{m}\left(Z_{\uparrow}^{(2)}\left(\mathbf{p} - \frac{\mathbf{k}}{2}\right) - Z_{\downarrow}^{(2)}\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right)\right) \tag{25}$$

as well as

$$\begin{aligned}
 \delta F^{(3)}_{ik} &= \frac{\partial \tilde{\Gamma}_{\uparrow, \downarrow}^{(2)}(K; P, P')}{\partial p_0} \Bigg|_{P=P^{(0)}_i} P^{(1)}_k \\
 &\quad + \frac{\partial \tilde{\Gamma}_{\uparrow, \downarrow}^{(2)}(K; P, P')}{\partial p'_0} \Bigg|_{P'=P^{(0)}_k} P'^{(1)}_i
 \end{aligned} \tag{26}$$

in Eqs. (17), (23) are the only ones responsible for the differences between  $W_{\uparrow}$  and  $W_{\downarrow}$  and different components of  $F_{ik}$  in the third order in density when the set of equations (1) does not collapse into a single equation (16). These terms are easily computed with the help of data on  $Z^{(2)}$  and  $\tilde{\Gamma}^{(2)}$  in Appendices A, B.

Now let us return to Eq. (16) which we will rewrite, with the same accuracy as Eq. (24), in the form

$$\left[ \delta\omega - \mathbf{k} \frac{\mathbf{v}_\uparrow + \mathbf{v}_\downarrow}{2} + 2(\beta - \beta_1(\mathbf{p}))H \right] \delta n_{\uparrow\downarrow}(\mathbf{p}) \\ = \int \tilde{\Gamma}_{\uparrow\downarrow, \uparrow\downarrow}(K; P_\uparrow^{(0)}, P_\downarrow^{(0)}) \delta n_{\uparrow\downarrow}(\mathbf{p}') (\theta_\downarrow(\mathbf{p}') - \theta_\uparrow(\mathbf{p}')) \frac{d^3 p'}{(2\pi)^3} \quad (27)$$

where we used the fact that components of the generalized interaction functions  $F_{ik}$  in the first and second order in density are equal to each other and are given by the irreducible vertex, Eq. (15) (the neglected terms in Eq. (27) are of the higher orders in density). On the other hand, in case of a homogeneous precession,  $\mathbf{k} = \delta\omega = 0$ , one has (cf. Eq. (I.51))

$$2(\beta - \beta_1(\mathbf{p}))H = \int \tilde{\Gamma}_{\uparrow\downarrow, \uparrow\downarrow}^{(2)}(K_0; P_\downarrow^{(0)}, P_\uparrow^{(0)}) (\theta_\downarrow(\mathbf{p}') - \theta_\uparrow(\mathbf{p}')) \frac{d^3 p'}{(2\pi)^3} \quad (28)$$

with  $K_0 = (2\beta H, \mathbf{0})$ .

We are interested in the eigenvalue of Eq. (27),  $\delta\omega$ , up to the terms of the order  $k^2$ ,  $\delta\omega = (\alpha^{(1)} + \alpha^{(2)})k^2$ . With this accuracy, we can restrict ourselves only to the linear in  $\mathbf{k}$  term in the expansion for  $\delta n_{\uparrow\downarrow}$  (here, as always, it is enough to know the eigenvectors in the first order if one is interested in the eigenvalues up to the second order):  $\delta n_{\uparrow\downarrow}(\mathbf{p}) = 1 + \mathbf{k}\mathbf{p} \delta v(\mathbf{p})$ . Then integration of Eq. (27) over momenta with an additional factor  $[\theta_\uparrow(\mathbf{p}) - \theta_\downarrow(\mathbf{p})]$  yields

$$\delta\omega(N_\uparrow - N_\downarrow) - \int \left( \mathbf{k} \frac{\mathbf{v}_\uparrow + \mathbf{v}_\downarrow}{2} \right) (\mathbf{k}\mathbf{p}) \delta v(\mathbf{p}) (\theta_\downarrow(\mathbf{p}) - \theta_\uparrow(\mathbf{p})) \frac{d^3 p}{(2\pi)^3} \\ = \int \tilde{\Gamma}_{\uparrow\downarrow, \uparrow\downarrow}(K; P_\uparrow^{(0)}, P_\downarrow^{(0)}) \mathbf{k}\mathbf{p}' \delta v(\mathbf{p}') (\theta_\downarrow(\mathbf{p}') - \theta_\uparrow(\mathbf{p}')) \\ \times (\theta_\downarrow(\mathbf{p}') - \theta_\uparrow(\mathbf{p}')) \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} \quad (29)$$

where one should put  $\mathbf{k} = 0$  in the arguments of the  $\theta$ -functions (3). As we will see below, the density expansions for  $\alpha$  and  $\delta v$  start from the term  $1/a$ . However, the expansion for the integral in the r.h.s. starts only from the term linear in  $a$ : the term with  $\tilde{\Gamma}^{(1)}$  (18) does not depend on  $k$  and vanishes after the angular integration, while the non-vanishing term  $\tilde{\Gamma}^{(2)}\delta v$  is linear in  $a$ . Since we are interested only in the first two density/interaction terms, Eq. (29) reduces to a simple expression

$$\alpha = \frac{1}{(N_\uparrow - N_\downarrow)} \int \frac{\mathbf{k}\mathbf{v}_\uparrow + \mathbf{k}\mathbf{v}_\downarrow}{2k} \frac{\mathbf{k}\mathbf{p}}{k} \delta v(\mathbf{p}) (\theta_\downarrow(\mathbf{p}) - \theta_\uparrow(\mathbf{p})) \frac{d^3 p}{(2\pi)^3} \quad (30)$$

while the eigenvector  $\delta v$  is equal to (see Appendix D)

$$\delta v^{(2)} = -\frac{1}{2m} (\delta(\varepsilon_{\downarrow}(\mathbf{p}) - \mu) + \delta(\varepsilon_{\uparrow}(\mathbf{p}) - \mu)) - \frac{1}{(4\pi a)^2 (N_{\uparrow} - N_{\downarrow})^2} \\ \times \int \frac{(\mathbf{p} - \mathbf{p}') \mathbf{k}}{\mathbf{p} \mathbf{k}} \tilde{\Gamma}_{\uparrow\downarrow, \downarrow\uparrow}^{(2)}(K_0; P^{(0)}, P'^{(0)}) (\theta_{\downarrow}(\mathbf{p}) - \theta_{\uparrow}(\mathbf{p})) \frac{d^3 p'}{(2\pi)^3}$$

Finally, the spectrum of spin waves (30) obtains the form

$$\omega = 2\beta H + \alpha k^2, \quad \alpha = \frac{(6\pi^2)^{2/3} N_{\uparrow}^{5/3} - N_{\downarrow}^{5/3}}{20\pi a m (N_{\uparrow} - N_{\downarrow})^2} + \alpha^{(2)} \\ \alpha^{(2)} = \alpha' + i\alpha'', \quad \alpha' = \frac{\pi}{6m\Delta} F_1(\Delta), \quad \alpha'' = -\frac{2^{7/3}\pi}{35m} F_2(\Delta) \quad (31)$$

(see Appendix D), where

$$\Delta \equiv \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$

is the degree of spin polarization. The functions  $F_{1,2}(\Delta)$  are given in Fig. 1 (cf. Refs. 10, 11). Note that  $\alpha''$  (31) does not contain  $a$  and is a universal function.

The divergence of the spin waves spectrum (31) at low polarizations,  $N_{\uparrow} - N_{\downarrow} \rightarrow 0$ , or at low density/interaction,  $aN^{1/3} \rightarrow 0$ , is purely formal and is insignificant. The spectrum is quadratic,  $\delta\omega \propto k^2$ , Eq. (31), only as far as the wave vector is small,  $k \ll a(N_{\uparrow} - N_{\downarrow})/N^{1/3}$  and becomes linear (or disappears completely because of the Landau damping; see Ref. 9) at higher wave vectors. This inequality effectively removes the singularity at  $a(N_{\uparrow} - N_{\downarrow})/N^{2/3} \rightarrow 0$ . In principle, such a formal singularity is present in some form in all expressions for the spectrum,<sup>3-6,9</sup> and reflects a simple fact that it is impossible to have a transition from a quadratic spectrum in polarized systems to a linear spectrum in non-polarized systems without a formal singularity at  $a(N_{\uparrow} - N_{\downarrow})/N^{2/3} \rightarrow 0$ .

The imaginary part of the coefficient  $\alpha$ , Eq. (31), gives the zero-temperature attenuation. As it was discussed in some detail in Ref. 1, the origin of this attenuation at  $T=0$  is not a hidden relaxation process, but rather an inhomogeneous broadening caused by a dephasing of inhomogeneous precessions for tilted spin-ups and spin-downs. This dephasing is explained by the fact that the effective molecular fields acting on inhomogeneously tilted spin-ups and spin-downs are somewhat different leading to a difference in precession frequencies.

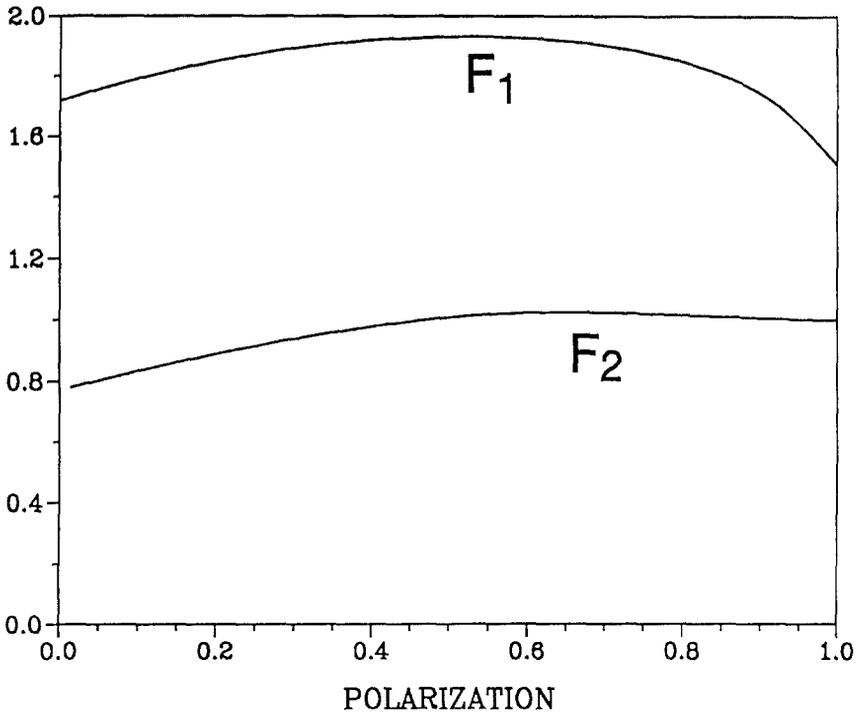


Fig. 1. Functions  $F_1$  and  $F_2$  for the spin wave spectrum  $\alpha'$  and  $\alpha''$  in the second order in density, Eqs. (31), (D.5), (D.6).

The appearance of the zero-temperature attenuation already in the second order in the density/interaction can also be easily understood from the following argument. As it was demonstrated in the previous Section, the spin dynamics equations (1) up to the second order in density collapse into a single equation (16). As a result, the corresponding transverse spin dynamics can be discussed not in terms of partial *pseudo*-densities, but in terms of a single transverse spin density  $\delta n_{\perp 1}$  which should include all interaction renormalizations up to the second order. Then the corresponding Landau interaction function in the second order can be given by a standard expression (cf. Ref. 7, 9),

$$\frac{32\pi^2 a^2}{m} \int \frac{d^3 p'_1}{(2\pi)^3} \frac{n_1(\mathbf{p}_1) + n_1(\mathbf{p}_2)}{p_1^2 + p_2^2 - p_1'^2 - (\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}'_1)^2 + i0 \operatorname{sign}(p_1 - p_1')} \quad (32)$$

The imaginary part of this integral is zero in case of zero polarization, or, for spin-polarized systems, for longitudinal processes. Therefore, the term  $i0$  in the denominator (31) is usually neglected. The situation is different in

case of transverse processes in spin-polarized systems: then the imaginary (pole) part in Eq. (32) is non-zero and is equal to

$$\begin{aligned}
 & -\frac{8a^2}{m} \int \{n_{\downarrow}(\mathbf{p}_1) n_{\uparrow}(\mathbf{p} + \mathbf{p}' - \mathbf{p}_1) + (1 - n_{\downarrow}(\mathbf{p}_1))(1 - n_{\uparrow}(\mathbf{p} + \mathbf{p}' - \mathbf{p}_1))\} \\
 & \quad \times \delta(p^2 + p'^2 - p_1^2 + (\mathbf{p} + \mathbf{p}' - \mathbf{p}_1)^2) d^3p_1 \quad (33)
 \end{aligned}$$

(see Appendix A). Detailed calculation of the cumbersome integral (33) is given in Appendix A. If one uses the Landau interaction function with the imaginary part (33) (and the corresponding equations from Appendix A) for the standard equations of the Fermi liquid theory,<sup>6,9</sup> one will get exactly the same value of the zero-temperature attenuation as in Eq. (31). The term with  $i0$  in the denominator (32) plays the role of the source of the collisionless Landau damping. This pole was missing in all previous calculations in the second order.

From the practical point of view this means that one can still use all the general equations of the Landau theory of low-density spin-polarized Fermi liquids,<sup>9</sup> but with one important modification: the transverse part of the Landau interaction function (A.3) should include, in addition to the expressions of Ref. 9, an imaginary part (33), (A.7-11).

Though the main purpose of this paper is the description of transverse spin dynamics and not of longitudinal effects (which obey a conventional Landau theory), our calculations of the vertex and the mass operators allow us to calculate the interaction renormalizations for the thermodynamic parameters. Therefore, as a by-product, we calculated the second-order

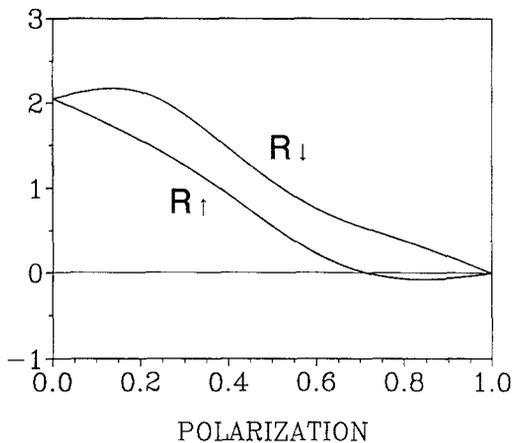


Fig. 2. Renormalization functions  $R_{\uparrow}$  and  $R_{\downarrow}$  for the effective masses in the second order in density, Eqs. (34), (C.2).

density corrections for the effective masses for spin-ups and spin-downs which are determined by the self-energy operators for the pure spin states in the second order, Eq. (21) (see Appendix C):

$$\begin{aligned} \frac{1}{m_{\uparrow}^*} &\equiv \frac{1}{p_{\uparrow}} \frac{\partial \varepsilon_{\uparrow}(p=p_{\uparrow})}{\partial p} = \frac{1}{m} + \frac{(p_0 a)^2}{\pi^2} R_{\uparrow}(\Delta) \\ \frac{1}{m_{\downarrow}^*} &\equiv \frac{1}{p_{\downarrow}} \frac{\partial \varepsilon_{\downarrow}(p=p_{\downarrow})}{\partial p} = \frac{1}{m} + \frac{(p_0 a)^2}{\pi^2} R_{\downarrow}(\Delta) \end{aligned} \quad (34)$$

The functions  $R_{1,2}(\Delta)$  are plotted in Fig. 2. At zero polarization these results coincide with Ref. 12.

#### 4. EFFECTS OF NON-LOCALITY IN TRANSVERSE SPIN DYNAMICS

Another important source of the attenuation is the non-locality of the particle interaction. This source might be especially important for the spin dynamics in  $^3\text{He}$ - $^4\text{He}$  mixtures where  $^3\text{He}$ - $^3\text{He}$  interaction is to a large extent mediated by phonons with long mean free paths thus increasing the range of interaction.

The non-locality corresponds to a dependence of the exact vertex part  $\tilde{\Gamma}$  on the momentum transfer  $\mathbf{k}$ . The irreducible vertex part can be expressed through the scattering amplitude as

$$\begin{aligned} &\tilde{\Gamma}(2\beta H, \mathbf{k}; \omega, \mathbf{p}, \omega', \mathbf{p}') \\ &= \frac{4\pi}{m} f(\mathbf{k}; \mathbf{p}, \mathbf{p}') - \left(\frac{4\pi}{m}\right)^2 \int \frac{d^3 q}{(2\pi)^3} \\ &\quad \times f(\mathbf{p} - \mathbf{q} + \mathbf{k}/2; \mathbf{p}/2 + \mathbf{q}/2 + \mathbf{k}/4, \mathbf{p}' + \mathbf{p}/2 - \mathbf{q}/2 - \mathbf{k}/4) \\ &\quad \times f(\mathbf{q} - \mathbf{p} + \mathbf{k}/2; \mathbf{p}/2 + \mathbf{q}/2 - \mathbf{k}/4, \mathbf{p}' + \mathbf{p}/2 - \mathbf{q}/2 + \mathbf{k}/4) \\ &\quad \times \left[ \frac{1 - n_{\uparrow}(\mathbf{q}) - n_{\downarrow}(\mathbf{p} + \mathbf{p}' - \mathbf{q})}{\omega_1 + \omega_2 + \mu_{\uparrow} + \mu_{\downarrow} - (1/2m)[q^2 + (\mathbf{p} + \mathbf{p}' - \mathbf{q})^2] + i0 \operatorname{sign}(q - p_{\uparrow})} \right. \\ &\quad \left. - P \frac{2m}{p^2 + p'^2 - q^2 - (\mathbf{p} + \mathbf{p}' - \mathbf{q})^2} \right] \end{aligned} \quad (35a)$$

where the bare scattering amplitude  $f(\mathbf{k}; \mathbf{p}, \mathbf{p}') \equiv f(\mathbf{p} + \mathbf{k}/2, \mathbf{p}' - \mathbf{k}/2; \mathbf{p} - \mathbf{k}/2, \mathbf{p}' + \mathbf{k}/2)$  depends on the transferred momentum  $\mathbf{k}$  and the relative momentum of colliding particles  $\mathbf{p} - \mathbf{p}'$ . In the pure  $s$ -wave approximation without any non-localities,  $f = -a$ .

There are two sources of non-locality. The first one is associated with the finite range of the direct particles' interaction, while the second one is

caused by the non-locality and retardation of the phonon-mediated part of the interaction.

The non-locality in the direct interaction channel assumes that the range of the interaction  $r_0$  is large in comparison with the scattering length  $a$ . Then the density expansion for the amplitude has the form

$$f(\mathbf{k}; \mathbf{p}, \mathbf{p}') = -\frac{a}{1 + |\mathbf{p} - \mathbf{p}'|^2 r_0^2 + i|\mathbf{p} - \mathbf{p}'| a} \quad (35b)$$

In this paper we are not interested in non-local renormalizations of the real part of the spin-wave spectrum; we will calculate only the main non-local contributions to the zero-temperature attenuation leaving only the largest term in density in the expression for the real part  $\alpha'$  (31).

There are two types of direct non-local contributions. The largest comes from the terms  $\Sigma_{\uparrow\uparrow}(p_0 - \beta H, \mathbf{p}) - \Sigma_{\downarrow\downarrow}(p_0 + \beta H, \mathbf{p})$  and  $\tilde{\Gamma}_{\uparrow\downarrow, \downarrow\uparrow}(2\beta H, \mathbf{0}; P, P')$  [ $G_{\downarrow\downarrow}(p_0 + \beta H, \mathbf{p}) - G_{\uparrow\uparrow}(p_0 - \beta H, \mathbf{p})$ ] in Eq. (19) with the vertex part (35). Though we will not present here the detailed calculations, it is worth mentioning that the resulting expression for  $\alpha''$  differs from the integral (D.4) by the additional factor  $r_0^2(\mathbf{p}_1 - \mathbf{p}_2)^2$  in the integrand

$$\alpha''_{nl} = \frac{2^{10/3} \pi p_0^2 r_0^2}{35m} F_3(\Delta) \quad (36)$$

where the function  $F_3(\Delta)$  is given in Fig. 3 and Appendix E. This term in attenuation is small in comparison with  $\alpha''$  (31) to the extent  $r_0^2 p_1^2 \ll 1$ . Note, that Eq. (36) as well as Eq. (31) do not contain the scattering length  $a$  in the expressions for the attenuation.

There are also non-local terms which originate from the derivatives of the vertex function  $\tilde{\Gamma}$ . Though such non-localities play a dominant role at high temperatures, Ref. 13, their contribution to the zero-temperature attenuation is smaller than (36).

Generally, the non-local contribution to the zero-temperature attenuation is not negligible for dilute Fermi systems only if the interaction range  $r_0$  is larger than the scattering length  $a$ . This may be true for  $^3\text{He}$  quasi-particles in  $^3\text{He}$ - $^4\text{He}$  mixtures: the  $^3\text{He}$ - $^3\text{He}$  interaction is mediated by long-lived phonons and has a relatively large range for slow  $^3\text{He}$  particles, while the force constants and, therefore, the scattering length  $a$ , are small because of not very large difference between  $^3\text{He}$  and  $^4\text{He}$  atoms. The simplest approximation for the phonon-mediated interaction vertex is

$$\tilde{\Gamma}_{ph}(Q; P, P') = \frac{4\pi a^* c^2 q^2}{m(q_0^2 - c^2 q^2 + i0)} \quad (38)$$

where the  $^3\text{He}$ -phonon interaction parameter  $a^* \approx -(0.1 \div 0.3) \text{ \AA}$  (see, e.g.,<sup>14</sup>),  $c \approx 2.36 \times 10^4 \text{ cm/s}$  is the sound velocity in  $^4\text{He}$ ,  $m \approx 2.3 m_3$  is the

effective mass of  $^3\text{He}$  quasi-particles,  $P$  and  $P'$  are the incoming and outgoing 4-momenta of  $^3\text{He}$  quasi-particles, and  $Q = (q_0, \mathbf{q})$  is the 4-momentum transfer corresponding to the internal phonon line for the two-particle vertex function. The interaction parameter  $a^*$  in Eq. (38) is written in such a way so that to provide a phonon contribution to the  $s$ -wave scattering length  $a$ .

Since we are interested in the case of low densities of  $^3\text{He}$  fermions, the interaction (38) can be treated perturbatively with  $v_F/c \sim aN^{1/3}$  serving as an additional expansion parameter ( $v_F$  is the Fermi velocity). Then, in the lowest orders, the phonon contribution to the attenuation of spin waves is given by the equation

$$\alpha''_{ph} = \frac{9\pi^2 a^* p_F^2}{35am^3c^2} F_4(\Delta) \quad (39)$$

(see Fig. 3 and Appendix E). Though this contribution is small, it might be important for a better experimental evaluation of the scattering length  $a$ .

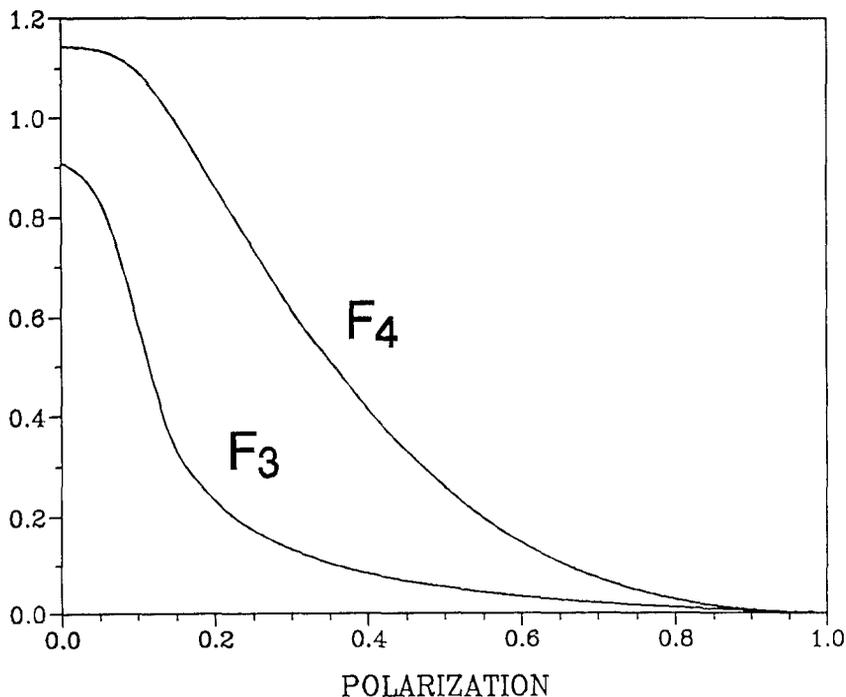


Fig. 3. Functions  $F_3$  and  $F_4$  for the nonlocal contributions to the zero temperature attenuation for the finite range potential  $\alpha_{nl}''$ , Eqs. (36), (E.6), and phonon mediated interaction  $\alpha_{ph}''$ , Eqs. (39), (E.3).

## 5. DISCUSSION

In this paper we applied our exact microscopic theory<sup>1</sup> of transverse phenomena in spin-polarized Fermi liquids to low-density Fermi systems such as a dilute gas of  $^3\text{He}$  quasi-particles in  $^3\text{He}\uparrow$ - $^4\text{He}$  liquid mixtures. For the first time, the results take consistently into account the inherent spatial and temporal non-locality of the fermion interaction (such as, for example, retardation). We calculated all the relevant quantities as an expansion in the fermion density. For degenerate Fermi systems such an expansion formally coincides with the expansion in the fermion interaction.

The spatial and, especially, temporal non-localities lead to a peculiar non-zero collisionless attenuation in *transverse* spin dynamics of polarized Fermi gases even at zero temperature. This zero-temperature attenuation is important when the degree of spin polarization is of the order of or larger than the ratio  $T/T_F$ .

As expected, the results in the lowest order in density and interaction coincide with the results of standard approaches which are based on the conventional Landau theory or transport equation.<sup>6,9</sup> Within this approximation, the exact irreducible vertex is always a constant, and does not contain any non-localities. Of course, within this accuracy, the zero-temperature attenuation vanishes.

The first unusual effects start to show up in the second order. Here the main equations still resemble those of the Landau theory, though the analog of the *transverse* component of the Landau interaction function has an imaginary part. These imaginary contributions come from off-shell terms in the exact vertex (or, if one uses a purely transport approach, from a pole in the interaction function which has the same origin as the collisionless Landau damping in plasma). This is the main source of the zero-temperature attenuation in spin-polarized low-density Fermi systems. With this accuracy, it is still possible to use the equations of the Landau theory of spin-polarized low-density Fermi liquids,<sup>9</sup> but the transverse component of the Landau interaction function of Ref. 9 should be supplemented by some imaginary part. We calculated this imaginary part explicitly. In the lowest order the results are similar to the variational results.<sup>10</sup>

The calculated value of the zero-temperature attenuation can be used for evaluation of the transverse relaxation time  $\tau_{\perp}$  at  $T=0$ , which in the main approximation has the following form<sup>15</sup>:

$$\tau_{\perp} = \frac{9}{20} \frac{\pi^2 \hbar^4}{a^2 \text{Im } \alpha} \frac{p_{\uparrow}^5 - p_{\downarrow}^5}{(p_{\uparrow}^3 - p_{\downarrow}^3)^3} \quad (40)$$

This transverse relaxation time  $\tau_{\perp}$  ( $T=0$ ) can be used either for kinetic equation in  $\tau$ -approximation, or for the macroscopic equations of spin dynamics (the Leggett equation, see Ref. 6). Since the transport equation up to the second order in density/interaction has the standard form, the Leggett equation is still valid.

In spin-echo experiments<sup>16-19</sup> one usually recovers the ratio of the quality factor  $\Omega_i\tau_{\perp}$  and the transverse diffusion coefficients  $D_{\perp}$  (notations from Ref. 6). This ratio can be expressed via the spectrum of spin waves as<sup>6</sup>

$$\frac{1}{\Omega_i\tau_{\perp}} = \frac{\alpha''}{\alpha'}, \quad \alpha' = \frac{D_{\perp}\Omega_i\tau_{\perp}}{1 + \Omega_i^2\tau_{\perp}^2} \quad (41)$$

The ratio  $\Omega_i\tau_{\perp}(T=0)/D_{\perp}(T=0)$  can be given, according to Eq. (31), as

$$\frac{\Omega_i\tau_{\perp}(T=0)}{D_{\perp}(T=0)} = \frac{3p_F am}{\pi T_F} \left( 1 + \frac{2}{3} p_F a F_1(\Delta) \right) \quad (42)$$

If taken into account, the second term in the brackets will slightly lower the absolute value of the scattering length  $a$  for  $^3\text{He}$ - $^4\text{He}$  mixtures which is given in Ref. 20.

The drastic changes occur in the next order in density/interaction. Here the result of the temporal non-locality is not only the zero-temperature attenuation, but the change in the form of the main equations themselves. With this accuracy, it is impossible to describe the transverse spin dynamics using any type of a closed transport equation in the fermion density as it is done in Ref. 10. The corresponding equation splits into two coupled equations in some complex *pseudo*-densities. The reason is the shift between different spin-up and spin-down shells for the vertex which is caused by retardation. In some sense, the effect reflects the spin-up—spin-down asymmetry which manifests itself only starting from this order in interaction. We calculated the leading terms which are responsible for this split of the equations.

We calculated all relevant microscopic characteristics (such as vertex functions, mass operators, etc.), and on their basis—the parameters of transverse spin dynamics, including the zero-temperature attenuation. As a by-product, we calculated also the effective masses of spin-up and spin-down fermions. The results include non-localities in direct, as well as in indirect, interaction channels. The latter one is especially important for  $^3\text{He}\uparrow$ - $^4\text{He}$  mixtures because of a strong retardation of a phonon-mediated part of interaction.

Needless to say, that all unconventional effects disappear for *longitudinal* spin dynamics or in a case of low spin polarization.

These results clearly demonstrate how and when the standard procedure of a derivation of the transport Boltzmann equation for degenerate spin-polarized Fermi gases breaks down. Later we plan to demonstrate this effect on the example of Kadanoff-Baym or Keldysh diagrammatic techniques of derivation of the transport equation for spin-polarized Fermi gases.

Another unclear question which should be studied separately, is the applicability of the macroscopic Leggett equation of spin dynamics at high spin polarizations in higher orders in density.

Some preliminary results have already been published in Ref. 15.

We are grateful to F. Laloe for helpful discussions. We are also grateful to D. Golosov for pointing out a numerical error in one of the results in Ref. 15. This work has been supported by NSF DMR-9100197 and INT-9015836.

## APPENDIX A

The irreducible vertex in the second order in the density expansion is given by a set of diagrams presented in Fig. 4 (cf. Refs. 7, 8). The corresponding analytical expression on the mass surface is:

$$\begin{aligned} \tilde{\Gamma}_{\uparrow\downarrow,\uparrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') &= -\frac{32\pi^2 a^2}{m} \int \left[ \frac{1 - n_{\downarrow}(\mathbf{p}_1) - n_{\uparrow}(\mathbf{p} + \mathbf{p}' - \mathbf{p}_1)}{p^2 + p'^2 - p_1^2 - (\mathbf{p} + \mathbf{p}' - \mathbf{p}_1)^2 + i0 \operatorname{sign}(p_1 - p_{\downarrow})} \right. \\ &\quad \left. - P \frac{1}{p^2 + p'^2 - p_1^2 - (\mathbf{p} + \mathbf{p}' - \mathbf{p}_1)^2} \right] \frac{d^3 p_1}{(2\pi)^3} \end{aligned} \quad (\text{A.1})$$

Isolating the real part of the irreducible vertex and integrating we get:

$$\begin{aligned} \operatorname{Re} \tilde{\Gamma}_{\uparrow\downarrow,\uparrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') &= \frac{2a^2}{m} \sum_{\uparrow=\uparrow,\downarrow} \left( \frac{q}{4} \ln \frac{p_{\uparrow} q + p_{\uparrow}^2 - \mathbf{p}\mathbf{p}'}{p_{\uparrow} q - p_{\uparrow}^2 + \mathbf{p}\mathbf{p}'} \right. \\ &\quad \left. - \frac{2p_{\uparrow}^2 - p^2 - p'^2}{4r} \ln \frac{p_{\uparrow} r + p_{\uparrow}^2 + \mathbf{p}\mathbf{p}'}{p_{\uparrow} r - p_{\uparrow}^2 - \mathbf{p}\mathbf{p}'} - p_{\uparrow} \right) \end{aligned} \quad (\text{A.2})$$

where  $r = |\mathbf{p} + \mathbf{p}'|$ ,  $q = |\mathbf{p} - \mathbf{p}'|$ . It is not surprising that this equation leads to a standard expression<sup>9</sup> for the Landau interaction function of a spin-polarized dilute Fermi gas in the second order in density. In notations of Ref. 9 (see Eq. (4.1.7) and below), a general Landau interaction function of

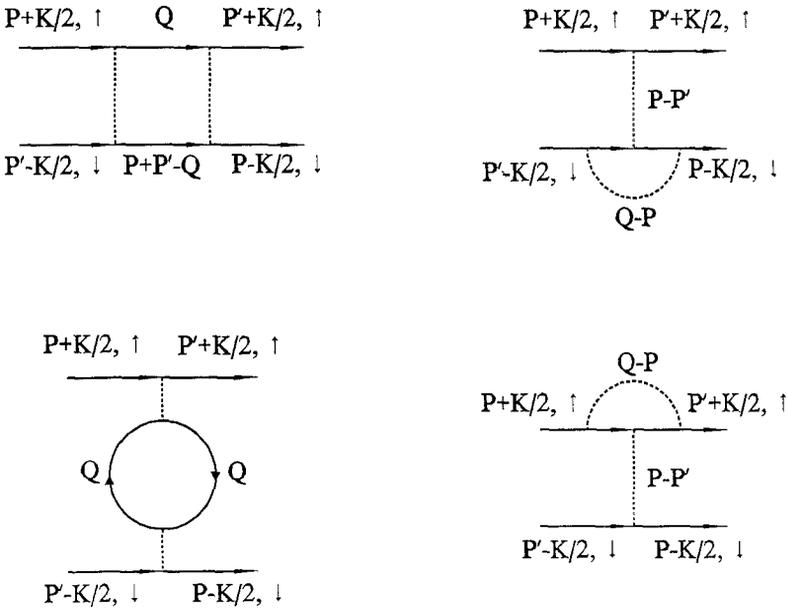


Fig. 4. Second order Feynman diagrams for the irreducible vertex  $\tilde{\Gamma}_{\uparrow\downarrow\uparrow}$ .

any polarized Fermi liquid of particles with spins 1/2 with an exchange interaction has the form

$$\begin{aligned}
 f_{\alpha\beta, \mu\nu}(\mathbf{p}, \mathbf{p}') &= \psi(\mathbf{p}, \mathbf{p}') \delta_{\alpha\beta} \delta_{\mu\nu} + \zeta(\mathbf{p}, \mathbf{p}') \sigma_{\alpha\beta} \cdot \sigma_{\mu\nu} \\
 &+ [\phi(\mathbf{p}, \mathbf{p}') \sigma_{\alpha\beta} \delta_{\mu\nu} + \phi(\mathbf{p}', \mathbf{p}) \delta_{\alpha\beta} \sigma_{\mu\nu}] \cdot \mathbf{e} \\
 &+ \xi(\mathbf{p}, \mathbf{p}') (\sigma_{\alpha\beta} \cdot \mathbf{e}) (\sigma_{\mu\nu} \cdot \mathbf{e})
 \end{aligned}
 \tag{A.3}$$

where the Greek indices denote the spin states, and  $\mathbf{e}$  is the unit vector in the direction of polarization. The transverse spin dynamics is described by the functions  $\zeta(\mathbf{p}, \mathbf{p}')$  and  $\xi(\mathbf{p}, \mathbf{p}')$ . According to direct standard calculations,<sup>9</sup> the function  $\zeta(\mathbf{p}, \mathbf{p}')$  in the second order in density/interaction is given exactly by Eq. (A.2), while  $\xi^{(2)} = 0$ . This equation is usually used for description of transverse spin dynamics in spin-polarized low-density Fermi liquids.

However, this does not provide the full description: the irreducible vertex (A.1) is complex with a noticeable imaginary part. The imaginary part of the integral (A.1) for the irreducible vertex on the mass surface is determined by  $i0$  in denominator and, therefore, is equivalent to the following integral with a  $\delta$ -function in the integrand:

$$\begin{aligned} \text{Im } \tilde{\Gamma}_{\uparrow\downarrow,\downarrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') &= \frac{4a^2}{m} \int \{ n_{\downarrow}(\mathbf{p}_1) n_{\uparrow}(\mathbf{p} + \mathbf{p}' - \mathbf{p}_1) \\ &\quad + (1 - n_{\downarrow}(\mathbf{p}_1))(1 - n_{\uparrow}(\mathbf{p} + \mathbf{p}' - \mathbf{p}_1)) \} \\ &\quad \times \delta(p^2 + p'^2 - p_1^2 - (\mathbf{p} + \mathbf{p}' - \mathbf{p}_1)^2) d^3p_1 \quad (\text{A.4}) \end{aligned}$$

Making a transformation  $\mathbf{s} = \mathbf{p}_1 - (\mathbf{p} + \mathbf{p}')/2$  and using spherical coordinates for  $\mathbf{s}$ , one can integrate (A.4) over  $d\mathbf{s}$ :

$$\begin{aligned} \text{Im } \tilde{\Gamma}_{\uparrow\downarrow,\downarrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') &= \frac{\pi a^2 q}{2m} \int \left\{ n_{\downarrow} \left( \frac{\mathbf{p} + \mathbf{p}'}{2} + \mathbf{s} \right) n_{\uparrow} \left( \frac{\mathbf{p} + \mathbf{p}'}{2} - \mathbf{s} \right) \right. \\ &\quad \left. + \left( 1 - n_{\downarrow} \left( \frac{\mathbf{p} + \mathbf{p}'}{2} - \mathbf{s} \right) \right) \left( 1 - n_{\uparrow} \left( \frac{\mathbf{p} + \mathbf{p}'}{2} + \mathbf{s} \right) \right) \right\} d \cos \theta \quad (\text{A.5}) \end{aligned}$$

Here  $\theta$  is the angle between  $\mathbf{p} + \mathbf{p}'$  and  $\mathbf{s}$ , and  $|\mathbf{s}| = q/2$ . To perform the remaining angular integration, one can formally take the density functions at finite temperature, and then take the limit  $T \rightarrow 0$  after integration. Integration at a finite temperature  $T$  yields (cf. Ref. 11):

$$\begin{aligned} \text{Im } \tilde{\Gamma}_{\uparrow\downarrow,\downarrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') &= \frac{2T\pi a^2}{r} \frac{1}{\exp \left[ \frac{1}{2mT} (p^2 + p'^2 - p_{\downarrow}^2 - p_{\uparrow}^2) \right] - 1} \\ &\quad \left( \left\{ \exp \left[ \frac{1}{4mT} (p^2 + p'^2 + rq - 2p_{\downarrow}^2) \right] + 1 \right\} \right. \\ &\quad \left. \times \left\{ \exp \left[ \frac{1}{4mT} (p^2 + p'^2 - 2p_{\uparrow}^2) \right] + \exp \left[ -\frac{rq}{4mT} \right] \right\} \right) \\ &= \times \ln \frac{\left( \left\{ \exp \left[ \frac{1}{4mT} (p^2 + p'^2 + rq - 2p_{\downarrow}^2) \right] + 1 \right\} \right.}{\left( \left\{ \exp \left[ \frac{1}{4mT} (p^2 + p'^2 - 2p_{\uparrow}^2) \right] + \exp \left[ \frac{rq}{4mT} \right] \right\} \right)} \quad (\text{A.6}) \end{aligned}$$

The limit  $T \rightarrow 0$  is reached differently for different regions of the variables  $\mathbf{p}$  and  $\mathbf{p}'$ . For the sake of simplicity we will consider only one region which is relevant to our calculations:  $p_{\downarrow} < p$ ,  $p' < p_{\uparrow}$  (in most of low frequency physical phenomena, the zero-temperature attenuation comes from the

region between the Fermi spheres for spin-ups and spin-downs). Other regions can be treated in the same way. Then the imaginary part of the irreducible vertex in the second order in the density expansion is given by the following set of equations:

$$p^2 + p'^2 < p_{\uparrow}^2, \quad |\cos \theta| < \frac{p_{\downarrow} \sqrt{p^2 + p'^2 - p_{\downarrow}^2}}{pp'};$$

$$\text{Im } \tilde{\Gamma}_{\uparrow\downarrow, \uparrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') = \frac{\pi a^2}{2rm} (2p_{\downarrow}^2 + rq - p^2 - p'^2) \quad (\text{A.7})$$

$$p_{\uparrow}^2 < p^2 + p'^2 < p_{\downarrow}^2 + p_{\uparrow}^2, \quad |\cos \theta| < \frac{p_{\uparrow} \sqrt{p^2 + p'^2 - p_{\uparrow}^2}}{pp'};$$

$$\text{Im } \tilde{\Gamma}_{\uparrow\downarrow, \uparrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') = \frac{\pi a^2}{rm} (p_{\downarrow}^2 + p_{\uparrow}^2 - p^2 - p'^2), \quad (\text{A.8})$$

$$p_{\uparrow}^2 < p^2 + p'^2 < p_{\downarrow}^2 + p_{\uparrow}^2$$

$$\frac{p_{\uparrow} \sqrt{p^2 + p'^2 - p_{\uparrow}^2}}{pp'} < |\cos \theta| < \frac{p_{\downarrow} \sqrt{p^2 + p'^2 - p_{\downarrow}^2}}{pp'};$$

$$\text{Im } \tilde{\Gamma}_{\uparrow\downarrow, \uparrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') = \frac{\pi a^2}{2rm} (2p_{\downarrow}^2 + rq - p^2 - p'^2) \quad (\text{A.9})$$

$$p^2 + p'^2 > p_{\downarrow}^2 + p_{\uparrow}^2, \quad |\cos \theta| < \frac{p_{\downarrow} \sqrt{p^2 + p'^2 - p_{\downarrow}^2}}{pp'};$$

$$\text{Im } \tilde{\Gamma}_{\uparrow\downarrow, \uparrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') = \frac{\pi a^2}{rm} (p^2 + p'^2 - p_{\downarrow}^2 - p_{\uparrow}^2) \quad (\text{A.10})$$

$$p^2 + p'^2 > p_{\downarrow}^2 + p_{\uparrow}^2, \quad \frac{p_{\downarrow} \sqrt{p^2 + p'^2 - p_{\downarrow}^2}}{pp'} < |\cos \theta| < \frac{p_{\uparrow} \sqrt{p^2 + p'^2 - p_{\uparrow}^2}}{pp'};$$

$$\text{Im } \tilde{\Gamma}_{\uparrow\downarrow, \uparrow\uparrow}^{(2)}(K; \mathbf{p}, \mathbf{p}') = \frac{\pi a^2}{2rm} (p^2 + p'^2 + rq - 2p_{\uparrow}^2) \quad (\text{A.11})$$

In all other parts of the region  $p_{\downarrow} < p$ ,  $p' < p_{\uparrow}$  the imaginary part of the vertex function is equal to zero.

In principle, with this accuracy, the transverse spin dynamics can be described by the equations of the Fermi liquid theory<sup>9</sup> with the interaction function (A.3) which should include, in addition to (A.2), the imaginary parts (A.7–11) in the transverse part of the Landau interaction function  $\zeta^{(2)}(\mathbf{p}, \mathbf{p}')$ , Eqs. (33), (A.3).

APPENDIX B

Interaction corrections to the single-particle energies in a dilute Fermi gas appear in the first order in the density expansion. However, these corrections do not depend on momenta,

$$\varepsilon_{\uparrow}^{(1)} = \frac{4\pi a \hbar^2}{m} N_{\downarrow}, \quad \varepsilon_{\downarrow}^{(1)} = \frac{4\pi a \hbar^2}{m} N_{\uparrow} \tag{B.1}$$

are always real, and do not lead to any deviations from a standard Fermi liquid behavior.<sup>6</sup>

Since the first-order corrections are momentum independent, the only second-order corrections to the single-particle energies are given by the self-energy operator in the second order taken at the frequency corresponding to the energy of a free particle:

$$\varepsilon_{\uparrow,\downarrow}^{(2)}(\mathbf{p}) = \Sigma_{\uparrow,\downarrow}^{(2)}\left(\frac{p^2}{2m} - \frac{p_{\uparrow,\downarrow}^2}{2m}, \mathbf{p}\right) \tag{B.2}$$

Here  $\Sigma^{(2)}$  is the part of the self-energy which ensures non-trivial contributions to the spectrum and which is given by the second order diagrams presented in Fig. 5. The rest of the second order diagrams contribute to the chemical potential  $\mu$  as well as to the spectrum, and, therefore are always cancelled from  $\varepsilon - \mu$ .

The analytical expressions for the diagrams in Fig. 5 are:

$$\begin{aligned} &\Sigma_{\uparrow\uparrow}^{(2)}(\omega, \mathbf{p}) \\ &= 2m \left(\frac{4\pi a}{m}\right)^2 \int \left\{ \frac{(1 - n_{\uparrow}(\mathbf{p}') - n_{\downarrow}(\mathbf{p} + \mathbf{q} - \mathbf{p}'))(n_{\downarrow}(\mathbf{q}) - n_{\uparrow}(\mathbf{p}'))}{2m\omega + p_{\uparrow}^2 + q^2 - p'^2 - (\mathbf{p} + \mathbf{q} - \mathbf{p}')^2 + i0 \operatorname{sign}(p' - p_{\uparrow})} \right. \\ &\quad \left. - P \frac{n_{\downarrow}(\mathbf{q})}{p^2 + q^2 - p'^2 - (\mathbf{p} + \mathbf{q} - \mathbf{p}')^2} \right\} \frac{d^3q d^3p'}{(2\pi)^6} \end{aligned} \tag{B.3}$$

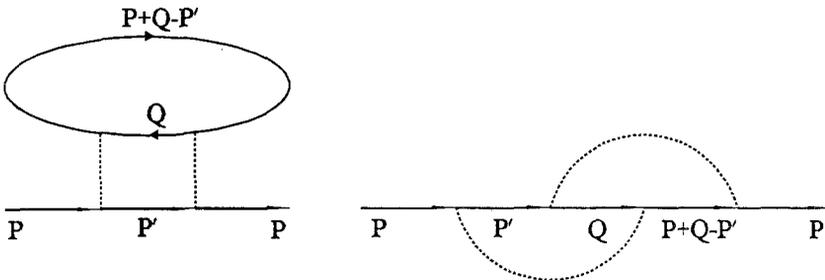


Fig. 5. Second order Feynman diagrams for the mass operator  $\Sigma$ .

and

$$\begin{aligned} & \Sigma_{\downarrow\downarrow}^{(2)}(\omega, \mathbf{p}) \\ &= 2m \left( \frac{4\pi a}{m} \right)^2 \int \left\{ \frac{(1 - n_{\downarrow}(\mathbf{p}') - n_{\uparrow}(\mathbf{p} + \mathbf{q} - \mathbf{p}'))(n_{\uparrow}(\mathbf{q}) - n_{\downarrow}(\mathbf{p}'))}{2m\omega + p_{\downarrow}^2 + q^2 - p'^2 - (\mathbf{p} + \mathbf{q} - \mathbf{p}')^2 + i0 \operatorname{sign}(p' - p_{\downarrow})} \right. \\ & \quad \left. - P \frac{n_{\uparrow}(\mathbf{q})}{p^2 + q^2 - p'^2 - (\mathbf{p} + \mathbf{q} - \mathbf{p}')^2} \right\} \frac{d^3 q d^3 p'}{(2\pi)^6} \end{aligned} \quad (\text{B.4})$$

First we will give the results for the imaginary part of energy (B.2-3); the corrections to the real part will be discussed in the end of this Appendix and in Appendix C. Using Eqs. (B.2-4), the imaginary part of the corrections to the spectra can be expressed as follows:

$$\begin{aligned} \operatorname{Im} \varepsilon_{\uparrow}(\mathbf{p}) &= -m \left( \frac{4\pi a}{m} \right)^2 \int \{ n_{\downarrow}(\mathbf{q})(1 - n_{\downarrow}(\mathbf{p} + \mathbf{q} - \mathbf{p}'))(1 - n_{\uparrow}(\mathbf{p}')) \\ & \quad - (1 - n_{\downarrow}(\mathbf{q})) n_{\downarrow}(\mathbf{p} + \mathbf{q} - \mathbf{p}') n_{\uparrow}(\mathbf{p}') \} \\ & \quad \times \delta(p^2 + q^2 - p'^2 - (\mathbf{p} + \mathbf{q} - \mathbf{p}')^2) \frac{d^3 q d^3 p'}{(2\pi)^5} \end{aligned} \quad (\text{B.5})$$

$$\begin{aligned} \operatorname{Im} \varepsilon_{\downarrow}(\mathbf{p}) &= -m \left( \frac{4\pi a}{m} \right)^2 \int \{ n_{\uparrow}(\mathbf{q})(1 - n_{\uparrow}(\mathbf{p} + \mathbf{q} - \mathbf{p}'))(1 - n_{\downarrow}(\mathbf{p}')) \\ & \quad - (1 - n_{\uparrow}(\mathbf{q})) n_{\uparrow}(\mathbf{p} + \mathbf{q} - \mathbf{p}') n_{\downarrow}(\mathbf{p}') \} \\ & \quad \times \delta(p^2 + q^2 - p'^2 - (\mathbf{p} + \mathbf{q} - \mathbf{p}')^2) \frac{d^3 q d^3 p'}{(2\pi)^5} \end{aligned}$$

As in Appendix A, it is easier to calculate these integrals for Fermi distributions  $n(\mathbf{p})$  at finite, but very small temperature, and then take the limit of  $T \rightarrow 0$ . Here, too, the calculation gives different results for different regions of momenta. We will not give the details of rather tedious, but simple calculations, and will present the results.

For  $\operatorname{Im} \varepsilon_{\uparrow}(\mathbf{p})$  with  $p < p_{\uparrow}$ , one has to consider two cases:  $p_{\uparrow} > p_{\downarrow} \sqrt{2}$  and  $p_{\uparrow} < p_{\downarrow} \sqrt{2}$ . For the former case calculations give the following expressions for three different ranges for variable  $p$ :

$$\begin{aligned} & p_{\uparrow} > \sqrt{2} p_{\downarrow}, \quad p < p_{\downarrow} \\ \operatorname{Im} \varepsilon_{\uparrow}(\mathbf{p}) &= \frac{a^2}{2\pi m} \left\{ \frac{1}{2} (p^2 - p_{\downarrow}^2)^2 + \frac{1}{3} (p_{\uparrow}^2 - p^2 - p_{\downarrow}^2)(p^2 + p_{\downarrow}^2) \right. \\ & \quad \left. + \frac{1}{p} \int_{\sqrt{p_{\uparrow}^2 - p^2}}^{p_{\uparrow}} q dq \left[ (y_{+\uparrow} - y_{-\uparrow})(p_{\uparrow}^2 + p_{\downarrow}^2 - p^2 - q^2) \right] \right\} \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{6} ((q+p)^3 - (q-p)^3 - y_{+\uparrow} + y_{-\uparrow}) \\
& + \frac{1}{2} (2p_{\downarrow}^2 - p^2 - q^2)(2p - y_{+\uparrow} + y_{-\uparrow}) \Big] \Big\} \\
& y_{+\uparrow} = \sqrt{p^2 + q^2 + 2p_{\uparrow}q \sqrt{p^2 + q^2 - p_{\uparrow}^2}} \\
& y_{-\uparrow} = \sqrt{p^2 + q^2 - 2p_{\uparrow}q \sqrt{p^2 + q^2 - p_{\uparrow}^2}}
\end{aligned} \tag{B.6a}$$

$$\begin{aligned}
\text{Im } \varepsilon_{\uparrow}(\mathbf{p}) = & \frac{a^2}{2\pi m p} \left\{ \int_{p_{\downarrow}}^{\sqrt{p_{\uparrow}^2 - p^2}} q dq \left[ \frac{1}{6} (y_{+\downarrow}^3 - y_{-\downarrow}^3) \right. \right. \\
& + \frac{1}{2} (2p_{\downarrow}^2 - p^2 - q^2)(y_{+\downarrow} - y_{-\downarrow}) \Big] \\
& + \int_{\sqrt{p_{\uparrow}^2 - p^2}}^{\sqrt{p_{\uparrow}^2 + p_{\downarrow}^2 - p^2}} q dq \left[ (p_{\uparrow}^2 + p_{\downarrow}^2 - p^2 - q^2)(y_{+\uparrow} - y_{-\uparrow}) \right. \\
& - \frac{1}{6} (y_{+\uparrow}^3 - y_{-\uparrow}^3 - y_{+\downarrow}^3 + y_{-\downarrow}^3) \\
& \left. \left. - \frac{1}{2} (2p_{\downarrow}^2 - p^2 - q^2)(y_{+\uparrow} - y_{-\uparrow} - y_{+\downarrow} + y_{-\downarrow}) \right] \Big\} \\
& y_{+\downarrow} = \sqrt{p^2 + q^2 + 2p_{\downarrow}q \sqrt{p^2 + q^2 - p_{\downarrow}^2}} \\
& y_{-\downarrow} = \sqrt{p^2 + q^2 - 2p_{\downarrow}q \sqrt{p^2 + q^2 - p_{\downarrow}^2}}
\end{aligned} \tag{B.6b}$$

$$\begin{aligned}
& p_{\uparrow} > \sqrt{2} p_{\downarrow}, \quad \sqrt{p_{\uparrow}^2 - p_{\downarrow}^2} < p < p_{\uparrow} \\
\text{Im } \varepsilon_{\uparrow}(\mathbf{p}) = & \frac{a^2}{2\pi m p} \int_{p_{\downarrow}}^{\sqrt{p_{\uparrow}^2 + p_{\downarrow}^2 - p^2}} q dq \left[ (p_{\uparrow}^2 + p_{\downarrow}^2 - p^2 - q^2)(y_{+\downarrow} - y_{-\downarrow}) \right. \\
& + \frac{1}{6} (y_{+\downarrow}^3 - y_{-\downarrow}^3 - y_{+\uparrow}^3 + y_{-\uparrow}^3) \\
& \left. + \frac{1}{2} (2p_{\downarrow}^2 - p^2 - q^2)(y_{+\downarrow} - y_{-\downarrow} - y_{+\uparrow} + y_{-\uparrow}) \right]
\end{aligned} \tag{B.6c}$$

In the second case, when  $p_{\uparrow} < p_{\downarrow} \sqrt{2}$ , integration gives the following expressions:

$$\begin{aligned}
 & p_{\uparrow} < \sqrt{2} p_{\downarrow}, \quad p < \sqrt{p_{\uparrow}^2 - p_{\downarrow}^2} \\
 \text{Im } \varepsilon_{\uparrow}(\mathbf{p}) = & \frac{a^2}{2\pi m} \left\{ \frac{1}{2} (p^2 - p_{\downarrow}^2)^2 + \frac{1}{3} (p_{\uparrow}^2 - p^2 - p_{\downarrow}^2)(p^2 + p_{\downarrow}^2) \right. \\
 & + \frac{1}{p} \int_{\sqrt{p_{\uparrow}^2 - p^2}}^{p_{\uparrow}} q dq \left[ (y_{+\uparrow} - y_{-\uparrow})(p_{\uparrow}^2 + p_{\downarrow}^2 - p^2 - q^2) \right. \\
 & + \frac{1}{6} ((q+p)^3 - (q-p)^3 - y_{+\uparrow} + y_{-\uparrow}) \\
 & \left. \left. + \frac{1}{2} (2p_{\downarrow}^2 - p^2 - q^2)(2p - y_{+\uparrow} + y_{-\uparrow}) \right] \right\} \quad (\text{B.7a}) \\
 & p_{\uparrow} < \sqrt{2} p_{\downarrow}, \quad \sqrt{p_{\uparrow}^2 - p_{\downarrow}^2} < p < p_{\downarrow}
 \end{aligned}$$

$$\begin{aligned}
 \text{Im } \varepsilon_{\uparrow}(\mathbf{p}) = & \frac{a^2}{2\pi m} \left\{ \frac{1}{2} (p^2 - p_{\downarrow}^2)^2 + \frac{1}{p} \int_{p_{\downarrow}}^{p_{\uparrow}} q dq \left[ (p_{\uparrow}^2 + p_{\downarrow}^2 - p^2 - q^2)(y_{+\uparrow} - y_{-\uparrow}) \right. \right. \\
 & + \frac{1}{2} (2p_{\downarrow}^2 - p^2 - q^2)(2p - y_{+\uparrow} + y_{-\uparrow}) \\
 & \left. \left. + \frac{1}{6} ((q+p)^3 - (q-p)^3 - y_{+\uparrow}^3 - y_{-\uparrow}^3) \right] \right\} \quad (\text{B.7b})
 \end{aligned}$$

$$\begin{aligned}
 & p_{\uparrow} < \sqrt{2} p_{\downarrow}, \quad p_{\downarrow} < p < p_{\uparrow} \\
 \text{Im } \varepsilon_{\uparrow}(\mathbf{p}) = & \frac{a^2}{2\pi m p} \int_{p_{\downarrow}}^{\sqrt{p_{\uparrow}^2 + p_{\downarrow}^2 - p^2}} q dq \left[ (p_{\uparrow}^2 + p_{\downarrow}^2 - p^2 - q^2)(y_{+\uparrow} - y_{-\uparrow}) \right. \\
 & + \frac{1}{2} (2p_{\downarrow}^2 - p^2 - q^2)(y_{+\downarrow} - y_{-\downarrow} - y_{+\uparrow} + y_{-\uparrow}) \\
 & \left. + \frac{1}{6} (y_{+\downarrow}^3 - y_{+\uparrow}^3 - y_{-\downarrow}^3 + y_{-\uparrow}^3) \right] \quad (\text{B.7c})
 \end{aligned}$$

When  $p > p_{\uparrow}$ , the final expressions for  $\text{Im } \varepsilon_{\uparrow}(\mathbf{p})$  are the same for both cases:

$$\begin{aligned}
 & p_{\uparrow} < p < \sqrt{p_{\uparrow}^2 + p_{\downarrow}^2} \\
 \text{Im } \varepsilon_{\uparrow}(\mathbf{p}) = & -\frac{a^2}{\pi m p} \left( \frac{1}{3} (p^2 - p_{\uparrow}^2 - p_{\downarrow}^2)(p_{\downarrow}^3 - (p_{\uparrow}^2 + p_{\downarrow}^2 - p^2)^{3/2}) \right. \\
 & \left. + \frac{1}{5} (p_{\downarrow}^5 - (p_{\uparrow}^2 + p_{\downarrow}^2 - p^2)^{5/2}) \right) \quad (\text{B.8})
 \end{aligned}$$

$$\begin{aligned}
 & p > \sqrt{p_{\uparrow}^2 + p_{\downarrow}^2} \\
 \text{Im } \varepsilon_{\uparrow}(\mathbf{p}) = & -\frac{a^2}{2\pi m} \left( \frac{1}{3} (p^2 - p_{\uparrow}^2 - p_{\downarrow}^2) p_{\downarrow}^3 + \frac{1}{5} p_{\downarrow}^5 \right)
 \end{aligned}$$

The remaining integrals (B.6–8) can be evaluated numerically:

$$\text{Im } \varepsilon_{\uparrow}(\mathbf{p}) = \frac{(p_0 a)^2}{2\pi m} F_{d\uparrow}(p), \quad p_0 = (3\pi^2(N_{\uparrow} + N_{\downarrow}))^{1/3} \quad (\text{B.9})$$

Functions  $F_{d\uparrow}(p)$  for three polarizations are given in Fig. 6a.

Analytical integration over all variables except for  $q$  gives the following expressions for  $\text{Im } \varepsilon_{\downarrow}(\mathbf{p})$  in four ranges of  $p$ :

$$p < p_{\downarrow} \\ \text{Im } \varepsilon_{\downarrow}(\mathbf{p}) = \frac{a^2}{4\pi m} (p_{\downarrow}^2 - p^2)^2 \quad (\text{B.10a})$$

$$p_{\downarrow} < p < p_{\uparrow} \\ \text{Im } \varepsilon_{\downarrow}(\mathbf{p}) = -\frac{a^2}{2\pi m p} \int_{\sqrt{p_{\uparrow}^2 + p_{\downarrow}^2 - p^2}}^{p_{\uparrow}} q dq \left[ (p_{\uparrow}^2 + p_{\downarrow}^2 - p^2 - q^2)(y_{+\downarrow} - y_{-\downarrow}) \right. \\ \left. + \frac{1}{2} (p^2 + q^2 - 2p_{\uparrow}^2)(y_{+\uparrow} - y_{-\uparrow} - y_{+\downarrow} + y_{-\downarrow}) \right. \\ \left. + \frac{1}{6} (y_{+\uparrow}^3 - y_{+\downarrow}^3 - y_{-\uparrow}^3 + y_{-\downarrow}^3) \right] \quad (\text{B.10b})$$

$$p_{\uparrow} < p < \sqrt{p_{\uparrow}^2 + p_{\downarrow}^2} \\ \text{Im } \varepsilon_{\downarrow}(\mathbf{p}) = -\frac{a^2}{2\pi m p} \left( \int_{p_{\downarrow}}^{p_{\uparrow}} q dq \left[ (p^2 + q^2 - p_{\uparrow}^2 - p_{\downarrow}^2)(y_{+\downarrow} - y_{-\downarrow}) \right. \right. \\ \left. \left. + \frac{1}{2} (p^2 + q^2 - 2p_{\uparrow}^2)(y_{+\uparrow} - y_{-\uparrow} - y_{+\downarrow} + y_{-\downarrow}) \right. \right. \\ \left. \left. + \frac{1}{6} (y_{+\uparrow}^3 - y_{+\downarrow}^3 - y_{-\uparrow}^3 + y_{-\downarrow}^3) \right] \right. \\ \left. + \frac{2}{3} (p^2 - p_{\uparrow}^2 - p_{\downarrow}^2)(p_{\downarrow}^3 - (p_{\uparrow}^2 + p_{\downarrow}^2 - p^2)^{3/2}) \right. \\ \left. + \frac{2}{5} (p_{\downarrow}^5 - (p_{\uparrow}^2 + p_{\downarrow}^2 - p^2)^{5/2}) \right) \quad (\text{B.10c})$$

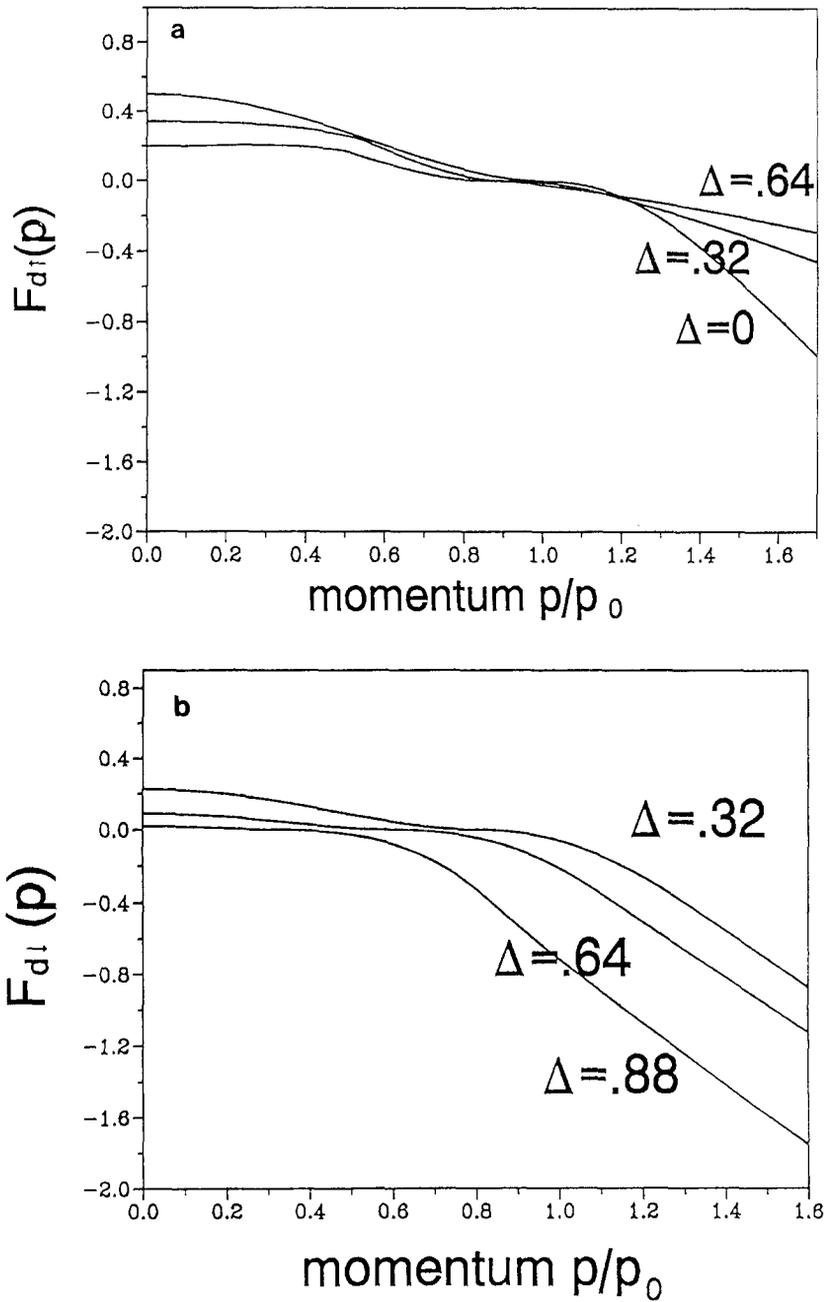


Fig. 6. Functions (a)  $F_{d\uparrow}$  Eq. (B.9), and (b)  $F_{d\downarrow}$  Eq. (B.11) for the imaginary part of the fermion spectra  $\text{Im } \varepsilon_{\uparrow,\downarrow}$  for three polarizations  $\Delta$ .

$$\begin{aligned}
 & p > \sqrt{p_{\uparrow}^2 + p_{\downarrow}^2} \\
 \text{Im } \varepsilon_{\downarrow}(\mathbf{p}) = & -\frac{a^2}{2\pi m p} \left\{ \int_{p_{\downarrow}}^{p_{\uparrow}} q \, dq \left[ (p^2 + q^2 - p_{\uparrow}^2 - p_{\downarrow}^2)(y_{+\downarrow} - y_{-\downarrow}) \right. \right. \\
 & + \frac{1}{2} (p^2 + q^2 - 2p_{\uparrow}^2)(y_{+\uparrow} - y_{-\uparrow} - y_{+\downarrow} + y_{-\downarrow}) \\
 & \left. \left. + \frac{1}{6} (y_{+\uparrow}^3 - y_{+\downarrow}^3 - y_{-\uparrow}^3 + y_{-\downarrow}^3) \right] \right. \\
 & \left. + p_{\downarrow}^3 \left( \frac{2}{3} (p^2 - p_{\uparrow}^2 - p_{\downarrow}^2) + \frac{2}{5} p_{\downarrow}^2 \right) \right\} \quad (\text{B.10d})
 \end{aligned}$$

Numerical integration over the remaining variable  $q$  yields:

$$\text{Im } \varepsilon_{\downarrow}(\mathbf{p}) = \frac{(p_0 a)^2}{2\pi m} F_{d\downarrow}(p) \quad (\text{B.11})$$

Functions  $F_{d\downarrow}(p)$  for three polarizations are given in Fig. 6b.

The real part of  $\varepsilon_{\uparrow,\downarrow}^{(2)}(\mathbf{p})$  is given by:

$$\begin{aligned}
 \text{Re } \varepsilon_{\uparrow}^{(2)}(\mathbf{p}) &= 2m \left( \frac{4\pi a}{m} \right)^2 \int P \frac{\left( n_{\uparrow}(\mathbf{p}') n_{\downarrow}(\mathbf{p} + \mathbf{q} - \mathbf{p}') \right.}{\left. - n_{\downarrow}(\mathbf{q})(n_{\uparrow}(\mathbf{p}') + n_{\downarrow}(\mathbf{p} + \mathbf{q} - \mathbf{p}')) \right)}{p^2 + q^2 - p^2 - (\mathbf{p} + \mathbf{q} - \mathbf{p}')^2} \frac{d^3 q \, d^3 p'}{(2\pi)^6} \\
 \text{Re } \varepsilon_{\downarrow}^{(2)}(\mathbf{p}) &= 2m \left( \frac{4\pi \alpha}{m} \right)^2 \int P \frac{\left( n_{\downarrow}(\mathbf{p}') n_{\uparrow}(\mathbf{p} + \mathbf{q} - \mathbf{p}') \right.}{\left. - n_{\uparrow}(\mathbf{q})(n_{\downarrow}(\mathbf{p}') + n_{\uparrow}(\mathbf{p} + \mathbf{q} - \mathbf{p}')) \right)}{p^2 + q^2 - p^2 - (\mathbf{p} + \mathbf{q} - \mathbf{p}')^2} \frac{d^3 q \, d^3 p'}{(2\pi)^6} \quad (\text{B.12})
 \end{aligned}$$

Analytical integration over  $\mathbf{q}$  gives the following intermediary equations:

$$\begin{aligned}
 \text{Re } \varepsilon_{\uparrow,\downarrow}^{(2)}(\mathbf{p}) &= 2m \left( \frac{4\pi a}{m} \right)^2 \int \left[ n_{\uparrow,\downarrow}(\mathbf{p}') \left\{ \frac{p_{\downarrow,\uparrow}^2 - u^2}{2} \ln \left| \frac{p_{\downarrow,\uparrow} + u}{p_{\downarrow,\uparrow} - u} \right| + u p_{\downarrow,\uparrow} \right\} \right. \\
 & \quad \left. + (n_{\uparrow}(\mathbf{p}') + n_{\downarrow}(\mathbf{p}')) \left\{ \frac{p_{\downarrow,\uparrow}^2 - v^2}{2} \ln \left| \frac{p_{\downarrow,\uparrow} + v}{p_{\downarrow,\uparrow} - v} \right| + v p_{\downarrow,\uparrow} \right\} \right] \\
 & \quad \times \frac{1}{|\mathbf{p} - \mathbf{p}'|} \frac{d^3 p'}{(2\pi)^5} \\
 & \quad u = \frac{p^2 - \mathbf{p} \cdot \mathbf{p}'}{|\mathbf{p} - \mathbf{p}'|}, \quad v = \frac{p'^2 - \mathbf{p} \cdot \mathbf{p}'}{|\mathbf{p} - \mathbf{p}'|} \quad (\text{B.13})
 \end{aligned}$$

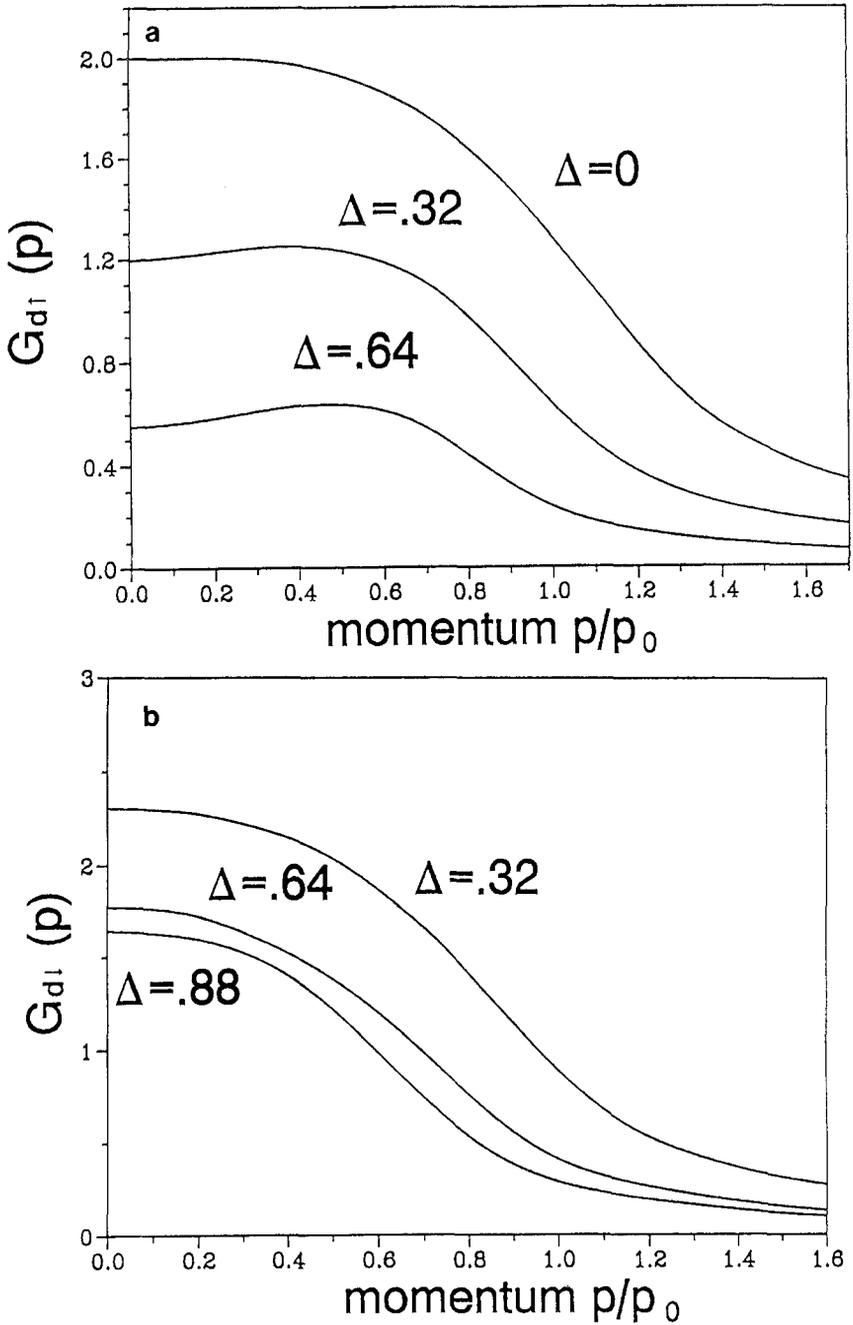


Fig. 7. Functions (a)  $G_{d\uparrow}$  and (b)  $G_{d\downarrow}$  for the real part of the fermion spectra  $\text{Re } \varepsilon_{\uparrow,\downarrow}$ , Eq. (B.14), for three polarizations  $\Delta$ .

The remaining integral over  $\mathbf{p}'$  can be evaluated numerically. The result for different polarizations is given by:

$$\text{Re } \varepsilon_{\uparrow,\downarrow}^{(2)}(\mathbf{p}) = \frac{1}{\pi^2} (p_0 a)^2 \frac{p_0^2}{m} G_{d\uparrow,\downarrow}(p) \quad (\text{B.14})$$

The functions  $G_{d\uparrow}(p)$  and  $G_{d\downarrow}(p)$  for three different polarizations are plotted on Figs. (7a and b).

In the second order the renormalization factor  $Z$  is given by:

$$Z_{\uparrow,\downarrow}(\mathbf{p}) = 1 + \left. \frac{\partial \Sigma_{\uparrow,\downarrow}^{(2)}(\omega, \mathbf{p})}{\partial \omega} \right|_{\omega = \varepsilon^{(0)}_{\uparrow,\downarrow}(\mathbf{p}) - \mu} \quad (\text{B.15})$$

Using Eqs. (2), (3) and performing the integration over  $\mathbf{q}$  in complete analogy with the previous case one obtains:

$$\begin{aligned} Z_{\uparrow,\downarrow} = & 1 + 2m^2 \left( \frac{4\pi a}{m} \right)^2 \int \left[ n_{\uparrow,\downarrow}(\mathbf{p}') \frac{d}{du} \left\{ \frac{p_{\downarrow,\uparrow}^2 - u^2}{2} \ln \left| \frac{p_{\downarrow,\uparrow} + u}{p_{\downarrow,\uparrow} - u} \right| + up_{\downarrow,\uparrow} \right\} \right. \\ & + (n_{\uparrow}(\mathbf{p}') + n_{\downarrow}(\mathbf{p}')) \frac{d}{dv} \left\{ \frac{p_{\downarrow,\uparrow}^2 - v^2}{2} \ln \left| \frac{p_{\downarrow,\uparrow} + v}{p_{\downarrow,\uparrow} - v} \right| + vp_{\downarrow,\uparrow} \right\} \left. \right] \\ & \times \frac{1}{|\mathbf{p} - \mathbf{p}'|} \frac{d^3 p'}{(2\pi)^5} \\ & u = \frac{p^2 - \mathbf{p} \cdot \mathbf{p}'}{|\mathbf{p} - \mathbf{p}'|}, \quad v = \frac{p'^2 - \mathbf{p} \cdot \mathbf{p}'}{|\mathbf{p} - \mathbf{p}'|} \end{aligned} \quad (\text{B.16})$$

The remaining integral over  $\mathbf{p}'$  can be evaluated numerically. The result for different polarizations is given by:

$$Z_{\uparrow,\downarrow}(\mathbf{p}) = 1 - \frac{2}{\pi^2} (p_0 a)^2 H_{d\uparrow,\downarrow}(p) \quad (\text{B.17})$$

The functions  $H_{d\uparrow}(p)$  and  $H_{d\downarrow}(p)$  for three different polarizations are plotted on Figs. (8a and b).

## APPENDIX C

Effective masses of spin-ups and spin-downs in the second order in the density expansion are given by:

$$\frac{1}{m_{\uparrow}^*} = \frac{1}{m} + \frac{1}{p_{\uparrow}} \frac{\partial \varepsilon_{\uparrow}^{(2)}(p = p_{\uparrow})}{\partial p}, \quad \frac{1}{m_{\downarrow}^*} = \frac{1}{m} + \frac{1}{p_{\downarrow}} \frac{\partial \varepsilon_{\downarrow}^{(2)}(p = p_{\downarrow})}{\partial p} \quad (\text{C.1})$$

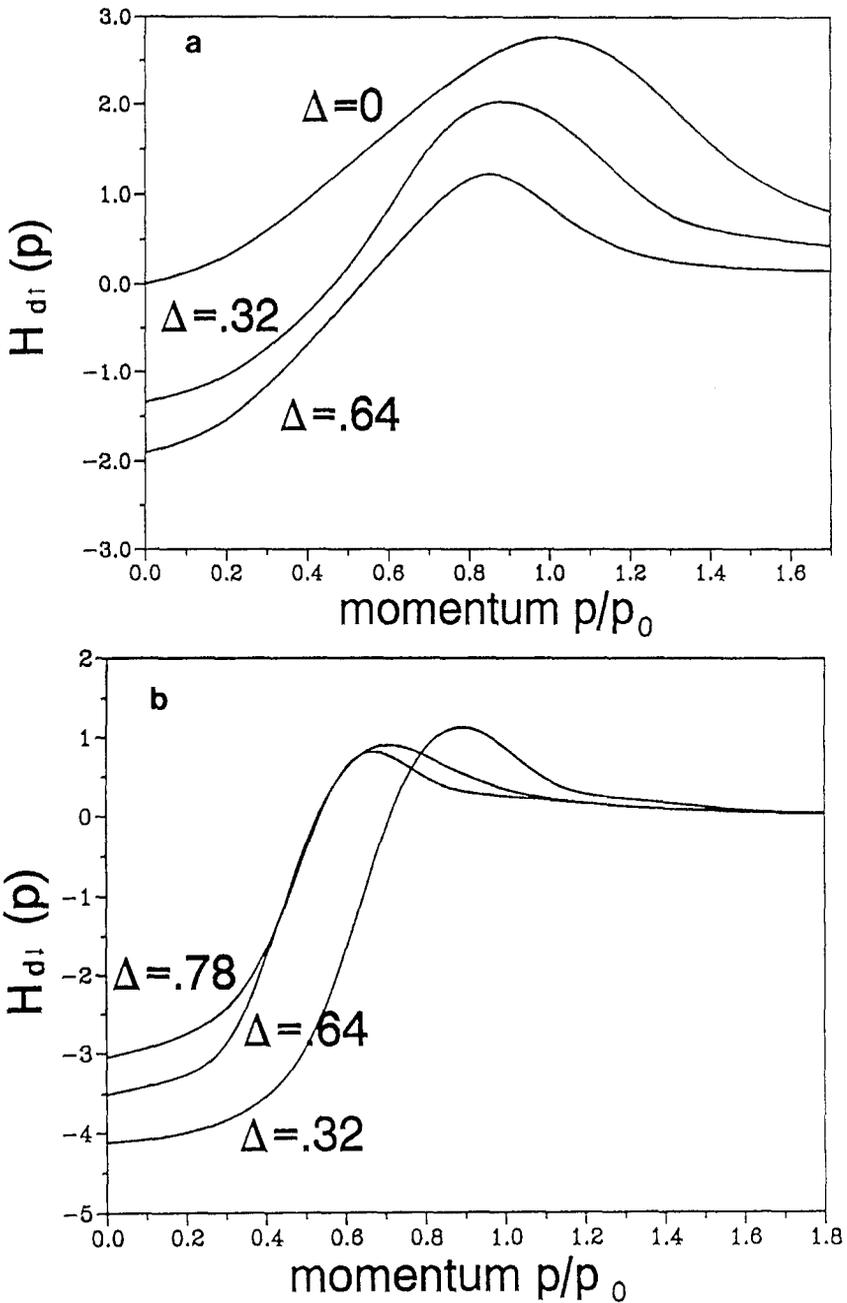


Fig. 8. Renormalization functions (a)  $H_{d↑}$  and (b)  $H_{d↓}$  for the factors  $Z_{↑,↓}$ , Eq. (B.17), for three polarizations  $\Delta$ .

Here  $m$  is the bare mass of the particle (effective mass of an isolated  ${}^3\text{He}$  atom in  ${}^3\text{He}$ - ${}^4\text{He}$  mixtures). Taking the derivative of the integrand (B.13) with respect to  $p$  and evaluating the integral numerically, one obtains:

$$\frac{1}{m_{\uparrow,\downarrow}^*} = \frac{1}{m} + \frac{(p_0 a)^2}{\pi^2} R_{\uparrow,\downarrow}(\Delta) \quad (\text{C.2})$$

Here  $\Delta$  is the polarization, and functions  $R_{\uparrow,\downarrow}(\Delta)$  are presented in Fig. 2.

### APPENDIX D

In the lowest order in the density expansion, the eigenvector  $\delta n = 1 + \mathbf{k}\mathbf{p} \delta v$  can be found from Eq. (19) by keeping only the terms which are of the first order in  $a$  and zeroth order in  $\mathbf{k}$ , and the terms which are of the first order in  $\mathbf{k}$  and the zeroth order in  $a$ . With this accuracy,

$$\delta v^{(1)} = -\frac{1}{4\pi a(N_{\uparrow} - N_{\downarrow})} \quad (\text{D.1})$$

This solution corresponds also to the lowest order expansion of Eq. (29). The next term in the expansion for the eigenvector  $\delta v$  can be obtained from the next order of the density expansion of Eq. (29) with the help of Eq. (D.1).

$$\begin{aligned} \delta v^{(2)} = & -\frac{1}{2m} (\delta(\varepsilon_{\downarrow}(p) - \mu) + \delta(\varepsilon_{\uparrow}(p) - \mu)) - \frac{1}{(4\pi a)^2 (N_{\uparrow} - N_{\downarrow})^2} \\ & \times \int \frac{(\mathbf{p} - \mathbf{p}')\mathbf{k}}{\mathbf{p}\mathbf{k}} \tilde{\Gamma}_{\uparrow\downarrow,\downarrow\uparrow}^{(2)}(K_0; P^{(0)}, P'^{(0)}) (\theta_{\downarrow}(\mathbf{p}) - \theta_{\uparrow}(\mathbf{p}')) \frac{d^3 p'}{(2\pi)^3} \quad (\text{D.2}) \end{aligned}$$

Substituting Eqs. (D.2) and (D.1) into Eq. (30) and separating the real and the imaginary parts, one obtains the following expressions for the spectrum  $\alpha^{(2)}$ :

$$\begin{aligned} \alpha' = & \frac{1}{m^2(N_{\uparrow} - N_{\downarrow})^3} \int \frac{d^3 p_1 d^3 p_2 d^3 p_3 d^3 p_4}{(2\pi)^9} \\ & \times \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) (\mathbf{p}_1 \cdot \mathbf{k}) ((\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{k}) \\ & \times P \frac{1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4} (n_{1\uparrow} n_{2\uparrow} (1 - n_{3\uparrow}) (1 - n_{4\downarrow}) \\ & + n_{1\uparrow} n_{2\downarrow} (1 - n_{3\downarrow}) (1 - n_{4\downarrow})) - \frac{1}{3m} \frac{(1 + \Delta)^{1/3} + (1 - \Delta)^{1/3}}{(1 + \Delta)^{1/3} - (1 - \Delta)^{1/3}} \quad (\text{D.3}) \end{aligned}$$

$$\begin{aligned}
\alpha'' &= \frac{1}{m^2(N_{\uparrow} - N_{\downarrow})^3} \int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^9} \\
&\times \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4)(\mathbf{p}_1 \cdot \mathbf{k})(\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{k} \\
&\times \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4)(n_{1\uparrow}n_{2\uparrow}(1 - n_{3\uparrow})(1 - n_{4\downarrow}) \\
&+ n_{1\uparrow}n_{2\downarrow}(1 - n_{3\downarrow})(1 - n_{4\downarrow}))
\end{aligned} \tag{D.4}$$

Numerical integration in Eq. (D.3) yields:

$$\alpha' = \frac{\pi}{6m\Delta} F_1(\Delta) \tag{D.5}$$

where  $\Delta = (N_{\uparrow} - N_{\downarrow})/(N_{\uparrow} + N_{\downarrow})$  is polarization, and function  $F_1(\Delta)$  is presented on Fig. 1.

A detailed analytical calculation of the integral in Eq. (D.4) one can find in Ref. 11. The result can be given as:

$$\alpha'' = -\frac{2^{7/3}\pi}{35m} F_2(\Delta) \tag{D.6}$$

with  $F_2(\Delta)$  presented on Fig. 1.

## APPENDIX E

Interaction of  $^3\text{He}$  impurities in  $^3\text{He}$ -HeII liquid mixtures is, to a large extent, mediated by phonons in superfluid HeII (see, for example, reviews<sup>6,9</sup>). The phonon (sound) velocity  $c$  is much higher than the characteristic Fermi velocity  $v_F$  of impurity particles which is proportional to the  $^3\text{He}$  concentration  $N^{1/3}$ . Therefore, in the main approximation the impurities can be considered as immobile (or the sound velocity as infinite), and phonons do not cause any retardation. The retardation effects caused by phonons appear in higher orders in  $v_F/c$ . The phonon-mediated interaction vertex (Eq. (38)) can be expanded in series of  $v_F/c$  as

$$\tilde{\Gamma}_{ph}(Q; P, P') = -\frac{4\pi a^*}{m} \left( 1 + \frac{q_0^2}{c^2 q^2} \right) \tag{E.1}$$

The parameterization of the vertex (E.1) is chosen so that the first (main) term in the brackets provides the phonon renormalization of the scattering amplitude, while the second term is responsible for the (small) retardation effects. We are interested in the latter, and assume that the scattering length

$a$  in all of the equations of this paper is already renormalized by the phonon-mediated interaction, i.e. that  $a$  already includes  $a^*$ .

Then the contribution to the imaginary part of the spectrum due to phonon-induced retardation can be evaluated by substituting the retardation term from Eq. (E.1) instead of the local vertex in the diagrams for the irreducible vertex part (Fig. 4). Simple calculations yield:

$$\begin{aligned} \alpha''_{ph} = & \frac{1}{4m^4c^2(N_{\uparrow} - N_{\downarrow})^3} \int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^9} \\ & \times \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4)(\mathbf{p}_1 \cdot \mathbf{k})(\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{k} \\ & \times \left( \frac{(p_1^2 - p_3^2)^2}{|\mathbf{p}_1 - \mathbf{p}_3|^2} + \frac{(p_2^2 - p_4^2)^2}{|\mathbf{p}_2 - \mathbf{p}_4|^2} \right) \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \\ & \times (n_{1\uparrow}n_{2\uparrow}(1 - n_{3\uparrow})(1 - n_{4\downarrow}) + n_{1\uparrow}n_{2\downarrow}(1 - n_{3\downarrow})(1 - n_{4\downarrow})) \quad (\text{E.2}) \end{aligned}$$

After numerical integration in (E.2), we get the following expression for the phonon contribution to the non-local part of the zero-temperature attenuation:

$$\alpha''_{ph} = \frac{9\pi^2 a^* p_0^2}{35m^3 c^2 a} F_4(\Delta) \quad (\text{E.3})$$

where the function  $F_4(\Delta)$  is plotted in Fig. 3.

In the same way, the non-local contribution to the vertex part in Eq. (31) can be written as

$$\tilde{\Gamma}_{nl}(Q; P, P') = \frac{4\pi a}{m} r_0^2 |\mathbf{p} - \mathbf{p}'|^2 \quad (\text{E.4})$$

The corresponding contribution to the attenuation takes the form:

$$\begin{aligned} \alpha''_{nl} = & \frac{2r_0^2}{m^2(N_{\uparrow} - N_{\downarrow})^2} \int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi)^9} \\ & \times \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4)(\mathbf{p}_1 \cdot \mathbf{k})(\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{k} \\ & \times |\mathbf{p}_1 - \mathbf{p}_2|^2 \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \\ & \times (n_{1\uparrow}n_{2\uparrow}(1 - n_{3\uparrow})(1 - n_{4\downarrow}) + n_{1\uparrow}n_{2\downarrow}(1 - n_{3\downarrow})(1 - n_{4\downarrow})) \quad (\text{E.5}) \end{aligned}$$

Numerical integration yields:

$$\alpha''_{nl} = \frac{2^{10/3}\pi p_0^2 r_0^2}{35m} F_3(\Delta) \quad (\text{E.6})$$

with the function  $F_3(\Delta)$  in Fig. 3.

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