Physica B 194–196 (1994) 845–846
North-Holland

PHYSICA B

Boundary effects in transverse spin dynamics of low temperature quantum gases

S. Stepaniants and A.E. Meyerovich

Department of Physics, University of Rhode Island, Kingston, RI 02881 - 0817, USA

We report the effects of boundary scattering by rough non-magnetic walls and exchange with adsorbed particles on spin dynamics of spin-polarized quantum gases. We derived a boundary condition with parameters expressed via the microscopic characteristics of the walls.

1. INTRODUCTION

Serious efforts are concentrated on lowering the temperature of $^3He\uparrow$-^4He mixtures. One of the crucial problems is thermometry since all equilibration times increase dramatically in a sub-mK range. A direct thermometry for a ^3He subsystem is to monitor temperature changes, in one of its characteristics, especially by NMR, with a known temperature dependence. At T< 1 mK mean free paths are very long, and all measurements should be accompanied by a thorough analysis of the boundary effects.

Often, boundary effects can be described by a general "hydrodynamic" boundary condition

$$\boldsymbol{M} + \Lambda n_i \nabla_i \boldsymbol{M} = 0, \qquad (1)$$

even if the particles are nearly ballistic (**M** is the magnetic moment, **n** is the unit vector normal to the boundary). The coefficient Λ contains the information about the scattering by non-magnetic walls: the scattering by surface inhomogeneities and the exchange with adsorbed particles.

1. SCATTERING BY A ROUGH WALL

The standard approach to scattering by rough walls is to use a microscopic boundary condition for the transport equation. This results in a very complicated integro-differential equation. We adopted another general method. We perform a non-linear coordinate transformation which transforms a rough wall into a smooth one, but adds random terms to the *bulk* Hamiltonian. We include these random bulk perturbations into the bulk collision integral. As a result, we reduce the transport problem of scattering by a rough wall to an equivalent transport problem with specular walls but with some stochastic bulk imperfections.

We consider a cell with one rough wall x=L-ξ(**s**) (**s** are the coordinates in the plane of the wall) and one specular wall, x=0. The function ξ(**s**) is random, <ξ>=0. The results contain a correlation function $\xi^{(2)}(|\mathbf{s}_1-\mathbf{s}_2|)=\langle\xi(\mathbf{s}_1)\xi(\mathbf{s}_2)\rangle$. The shift of inhomogeneity from the wall to the bulk is done by the transformation:

$$X' = \frac{xL}{L-\xi(s)};\quad Y' = y;\quad Z' = z \qquad (2)$$

"New" walls, X'=0 and X'=L, satisfy the simplest boundary condition Ψ(0)=Ψ(L)=0. In new variables the bulk Hamiltonian acquires stochastic perturbations which depend on ξ and are included into a perturbative collision integral as bulk impurities. This transport equation describes 2D diffusion along the wall

$$D_s(T) = \frac{8\pi L^2}{\alpha m N}\int d^3p \frac{p_\perp{}^2 M^{(0)}(p_x,p_\perp)}{\bar{\eta} p_\perp{}^4 + 4\bar{\xi} p_x{}^4} \qquad (3)$$

Here $M^{(0)}(\mathbf{p},T)$ is the equilibrium distribution of the magnetic moment,

$$\eta(p_\perp,\varphi) = \xi^{(2)}(p_\perp,\varphi)[1-\cos\varphi]^2,$$
$$\bar{\eta} = \eta_0 - \eta_1,\quad \bar{\xi} = \xi_0 - \xi_1$$

η_i and ξ_i are the harmonics of the functions η and $\xi^{(2)}$. By the order of magnitude,

$$D_s \approx \frac{\hbar}{m}(a^3N)^{-2/3}\left(\frac{aL}{\xi^{(2)}(0)R}\right)^2\frac{T_0}{E},$$

where R and $\xi^{(2)}(0)$ are the correlation radius and the height of surface inhomogeneities, N and T_0 are the density and the degeneracy temperature, and E is the larger of T and T_0. The parameter Λ in the boundary condition (1) is easily expressed via D_s (3).

2. EFFECTS OF BOUNDARY ADSORPTION IN SPIN DYNAMICS

Surface adsorption adds the exchange of adsorbed particles between themselves and with bulk particles. The former process is suppressed when the mobility of adsorbed particles and their density are low. In order to describe exchange collisions for identical bulk and adsorbed particles, we started from a general transport equation [1] for particles with free states and the states which are localized on some traps. The T-matrix for scattering of delocalized particles on localized ones contains the direct and exchange scattering, $t_1(\mathbf{p},\mathbf{p}')$ and $t_2(\mathbf{p},\mathbf{p}')$. For a short-range interaction $U(\mathbf{r})$, the exchange integral $t_2 \propto U(R_0/\lambda)^2$; in an opposite case, $t_2 \propto U(E/U_0)^2$ (R_0 and U_0 are the size and depth of the bound state, λ and $E<U_0$ are the particles' wavelength and energy). Collisions change the mean field near the wall, cause an additional dissipation, and result in a 2D spin diffusion along the wall ($p_T = (mT)^{1/2}$)

$$D_s = \frac{iT(\alpha N t_2(p_T, 0) - iw)}{m\hbar(\Omega_{int} + i/\tau_\perp)^2},$$

$$w = \frac{Np_T m}{(2\pi)^2\hbar^3}\int d\Omega[t_2^2+(t_1^2+t_1t_2)(1-\cos\theta)]$$

In a standard setting with a field gradient G, this leads to the attenuation of spin waves:

$$\omega'' = (q^2wG/\Omega)(\beta^{1/2}T/mG\Omega)^{2/3} + (1/3)(\gamma G)/\Omega^2)^{2/3}na^2(T/m)^{5/6} \quad (4)$$

We assume that the density of adsorbed particles is small, and their interaction is negligible. This is true for ^3He in mixtures when the adsorbed layer consists mostly of ^4He. With the lack of adequate data for helium, we applied our results to atomic hydrogen for which the density of the adsorbed layer rises exponentially at low temperatures. The agreement with data [2] is good at T > 0.15 K when the density of an adsorbed layer is low and our results are valid

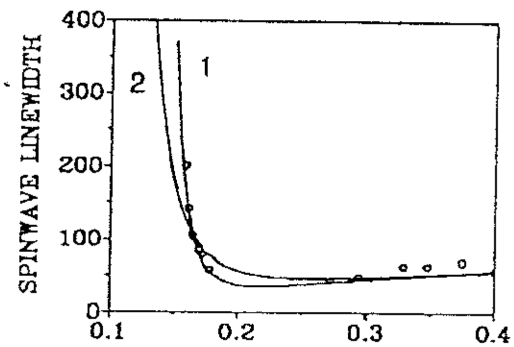

Figure 1. Linewidth for spin waves in hydrogen. Curve 1 and experimental data are taken from [2], curve 2 - Eq.(4).

4. SUMMARY

We analyzed effects of surface adsorption and inhomogeneities on spin dynamics. Both effects lead to spin diffusion along the wall. We derived an effective boundary condition which covers these two processes. The results are important at low temperatures when the mean free paths are long. Preliminary results were published in [3].

We thank G.Frossati and his group for hospitality and useful discussions. The work is supported by NSF DMR 9100197 and NATO CRG 910283.

REFERENCES

1. A.E.Meyerovich, Phys.Rev. B 39 (1989) 9318
2. N.P. Bigelow, J.H. Freed and D.M. Lee, Phys.Rev.Lett. 63 (1988) 1609
3. S. Stepaniants and A.E. Meyerovich, J.Low Temp.Phys. 89 (1992) 539