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Spin-Wave Instabilities in ^{3}He-^{4}He Solutions at High Magnetic Field

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During our recent high field pulsed NMR experiments on a spin-polarized saturated ^{3}He-^{4}He solution, we observed a long-lived oscillating signal in addition to the usual spin-echo. The oscillations were excited by the application of a single RF tipping pulse of angle greater than some critical value. They were obseravble only for temperatures below 20mK corresponding to values of the spin-rotation parameter $\mu M_0 \geq 3$. The signals were not present during experiments on lower concentration solutions. The frequencies of the oscillations were found to scale as the cosine of the tipping angle. The nature and shape of the oscillations suggest that they may be driven by instabilities through a mechanism proposed by Castaing. These instabilities arise due to a modification of the non-linear term in the Leggett equation by the large magnetization gradients present at the edges of the pulsed region.

1. INTRODUCTION

Studies of instabilities and non-linear effects help towards an understanding of spin dynamics in Fermi liquids. In this paper we discuss a new observation of instabilities in spin-polarized ^{3}He-^{4}He solutions.

During the course of our recent pulsed NMR experiments[1] on a saturated solution of ^{3}He in ^{4}He, we observed a peculiar oscillating signal in addition to the usual spin echo. Further investigation revealed that this signal could be induced by the application of a single RF tipping pulse. Below we discuss this non-linear ringing effect and explain it on the basis of an

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instability mechanism, similar to one proposed earlier by Castaing[2].

2. EXPERIMENTAL DETAILS

The experimental cell consisted of two Stycast chambers separated by a 1mm diameter Stycast tube. The upper chamber contained silver sinter in

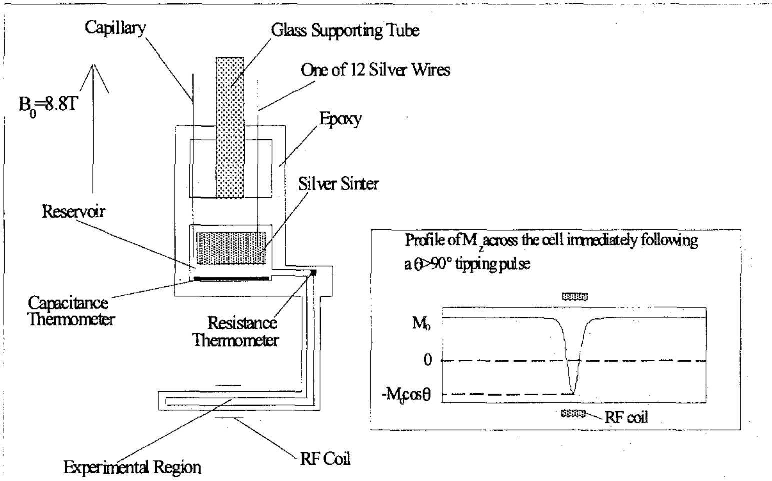

Fig. 1. The experimental cell

good thermal contact with the helium in the cell; the sinter was cooled via a silver thermal link to the mixing chamber of a dilution refrigerator. The lower chamber consisted of a 1mm inner diameter tube around which an rf coil (3mm radius, two turns of 0.6mm diameter Cu-wire) was positioned. A main field of 8.8T and a uniform gradient of 80mT/m were applied to the cell, in a direction normal to the axis of the tube (see Fig.1). The polarization of the saturated solution in such a field was a few percent and the Leggett spin rotation parameter μM_0 had a value of about 4 at the lowest temperatures. A single NMR pulse of varying length was applied to the experimental region of the cell; the spins are tipped through an angle θ in the region inside the rf coil.

3. RESULTS

A typical NMR signal produced by a $\theta = 105°$ pulse is plotted in Fig.2. This long-lived ringing was observed only when the tipping angle exceeded

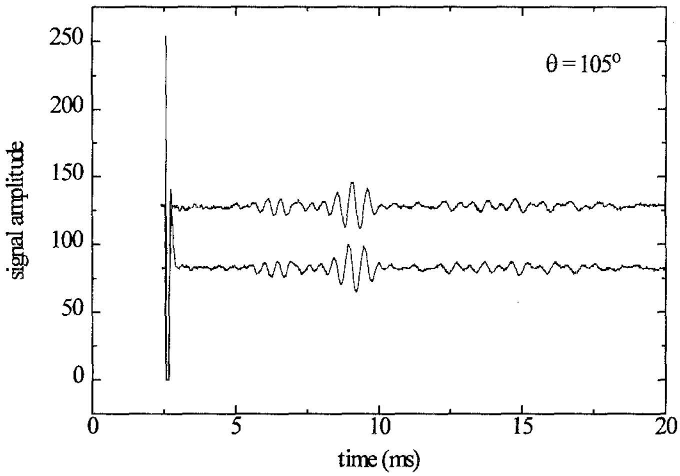

Fig. 2. The in-phase and quadrature signals observed following a $\theta = 105°$ tipping pulse

some critical value $\theta_c \simeq 70°$. The frequencies of the oscillations were determined by Fourier transforming the signals (Fig. 3); the frequency shift δf away from the Larmor frequency can be seen to increase with tipping angle.

The signals exhibited several important features. They appeared only at $\theta \geq \theta_c \sim 70°$, and were observable only for temperatures below 20mK, corresponding to a value of the spin rotation parameter $\mu M_0 \geq 3$. The timescale of the oscillations was long compared to that of a free induction decay and their frequencies were found to vary with the tipping angle (Fig. 3). The signals were not observed during the experiments on lower concentration solutions.

4. ANALYSIS: SPIN-WAVE INSTABILITIES

At low spin polarizations, the transverse spin dynamics in polarized Fermi liquids are governed by the Leggett equation[3] :

$$\frac{\partial \mathbf{M}}{\partial t} + [\gamma \mathbf{B} \times \mathbf{M}] = \frac{\partial}{\partial x_k}[\frac{D_\perp}{1+\mu^2 M_0^2}(\frac{\partial \mathbf{M}}{\partial x_k} + \mu[\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x_k}])] \qquad (1)$$

If the magnetization gradients are small, the last term can be linearized in small deviations from equilibrium $\delta\mathbf{M}$ as $\mu[\mathbf{M}_0 \times \partial\delta\mathbf{M}/\partial x_k]$, and the spin excitations correspond to weakly attenuated circularly polarized spin waves

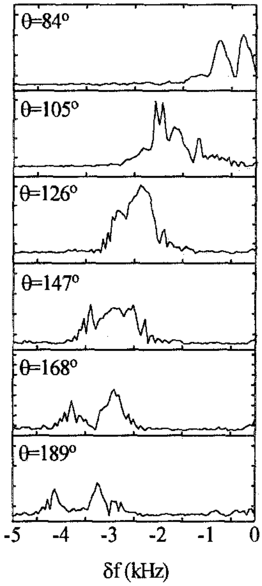

Fig. 3. The frequency spectra obtained by Fourier transforming the signals

with the spectrum

$$\omega = \omega_0 + \frac{D_\perp k^2}{1+\mu^2 M_0^2}(i - \mu M_0) \tag{2}$$

Castaing[2] noticed that if the gradient in the magnetization $\nabla\mathbf{M}$ is not negligible, the linearized last term in Eq. (1) is $\mu[\mathrm{M}_0 \times \partial\delta\mathbf{M}/\partial x_k]+\mu[\delta\mathbf{M}\times\nabla\mathrm{M}_0]$, and the excitation spectrum changes from Eq.(2) to

$$\omega = \omega_0 + \frac{D_\perp}{1+\mu^2 M_0^2}(i - \mu M_0)(k^2 - \mu\mathbf{k}\nabla\mathbf{M}). \tag{3}$$

For a sufficiently large magnetization gradient (or sufficiently small wavevectors), the last bracket and therefore the imaginary part of the spectrum changes sign. Instead of attenuation, the perturbation increases with time resulting in instability.

Large magnetization gradients were present in our experimental cell at the edges of the pulsed region (see Fig. 1). We can approximate these gradients, set up by the tipping pulse, as $\mid \nabla\mathrm{M} \mid \sim M_0(1-\cos\theta)/\Delta x$ where Δx is the distance over which they extend. This characteristic distance determines the typical wavevector k. The initial perturbation is provided by the transverse component of magnetization produced by the tipping pulse. When the tipping angle is increased beyond some critical value such that the magnetization gradient at the edge of the pulse becomes sufficiently large, these perturbations increase with time resulting in instability. This instability then propagates into the experimental region where it is detected by the receiving NMR coil. The effect is suppressed by the diffusion of up-spins into the region inside the coil; eventually the magnetization gradients are no longer sufficient to produce instability and the signals decay.

By substituting the expression for the magnetization gradient into the spectrum (Eq. 3) we find that the frequency, $\delta\omega = \omega - \omega_0$, depends upon tipping angle as $\delta\omega \propto \cos\theta - \cos\theta_c$, where θ_c is the critical angle; i.e. the angle for which the last bracket of the spectrum is equal to zero. A fit of this expression to the Fourier transform data is shown in Fig. 4. We have used Eq. 3 to estimate that the wavevector k of the instability produced is of order 600cm^{-1}. This implies that the large magnetization gradient is over a distance of order 0.05cm; this is consistent with the scale of our experimental setup.

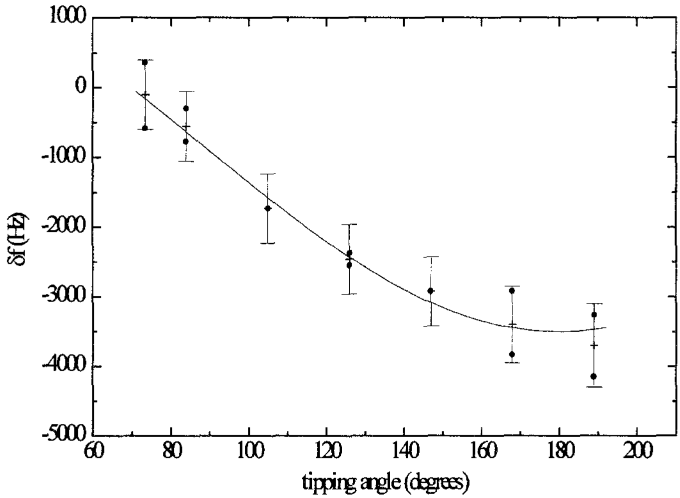

Fig. 4. The variation of the frequency of the signals with tipping angle. The points represent the largest peaks in the spectra while the bars represent the spread of frequencies. The line is a fit to $\delta f = A(\cos\theta - \cos\theta_c)$.

5. DISCUSSION

The signals which we observed possess several features which support an explanation in terms of an instability. The cutoff in tipping angle, together with the fact that there was no ringing signal at higher temperatures when μM_0 is small confirms the threshhold nature of the phenomenon. The long-time scale of the signals and the initial increase in amplitude (Fig.2) are also characteristic of an instability. The frequencies of the oscillations scaled as the cosine of the tipping angle, $\cos\theta - \cos\theta_c$. The presence of two frequency peaks (Fig.3) suggests that the signals are coming from regions either side of the RF coil where the magnetization gradients are slightly different. No such signals were observed during experiments on low concentration solutions. Here μM_0 is negative so that the instability propagates in the opposite direction, away from the receiving NMR coil.

Similar instabilities have been observed by Nunes[4] and recently by Dmitriev et al[5]. The ringing observed in these experiments continued for extremely long times, leading to conclusions about the existence of a metastable state (namely, precessing spin domains)[6] after the instability develops. In our experiment we did not see such a long-time behaviour, due to the different setup; only a small proportion of the spins in the lower chamber are tipped and the logitudinal spin diffusion coefficient $D_{\|}$ is large, so that the instability is quickly suppressed by diffusion of up-spins into the region inside the coil.

It would be interesting to look for a similar effect in low concentration solutions by positioning a receiving NMR coil further along the lower chamber of our experimental cell. Perhaps particularly interesting would be a study of the non-linear spin dynamics in solutions with ^{3}He concentrations approaching 4%; the point at which the propagation of the instability changes direction.

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