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# Surface Slip in Spin-Polarized and Binary Quantum and Classical Systems

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**Abstract**

We calculate accurate upper and lower bounds for slip coefficients in hydrodynamic boundary conditions for mass, spin, and thermal flows in polarized or binary classical and quantum gases at arbitrary polarizations and temperatures. The results have the most transparent form for high-temperature Boltzmann and low-temperature degenerate gases.

## Introduction

The large temperature and/or magnetization gradients, which are practically unavoidable [1–4] in experiments with systems in long-lived quasi-equilibrium polarized state, cause large slip, thermal, and mass flows through boundary layers. The thickness of those boundary layers, expressed via slip coefficients, is proportional to the mean free path $l$ and, therefore, increases dramatically with increasing spin polarization [5,6]. For the accurate interpretation of the existing experiments the estimates for all slip coefficients are prerequisite.

## Main equations

There are 9 coefficients $\alpha_{ik}$ in the slip boundary conditions:

$$\mathbf{u}_t(z=0) = \alpha_{11}\frac{\partial \mathbf{u}_t(z\to 0)}{\partial z} + \alpha_{12}\nabla_t\mu(z=0) + \alpha_{13}\nabla_t T(z=0),$$

$$\mathbf{J}_t^{(s)}(z=0) = \alpha_{21}\frac{\partial \mathbf{u}_t(z\to 0)}{\partial z} + \alpha_{22}\nabla_t\mu(z=0) + \alpha_{23}\nabla_t T(z=0),$$

$$\mathbf{Q}_t(z=0) = \alpha_{31}\frac{\partial \mathbf{u}_t(z\to 0)}{\partial z} + \alpha_{32}\nabla_t\mu(z=0) + \alpha_{33}\nabla_t T(z=0),$$

where $\mathbf{u}$ is the mass velocity, $\mathbf{J}^{(s)}$ and $\mathbf{Q}$ are the boundary slip diffusion and heat slip flows; the index $t$ marks the components of vectors along the boundary $z = 0$.

We estimated all of the coefficients $\alpha_{ik}$ separately, by solving the transport equation for each of the gradients $\nabla_x T(z)$, $\nabla_x \mu(z)$, $\nabla_x \mathbf{u}(z)$ and expressing $\mathbf{u}(z)$, $J^{(s)}$ and $Q^{(s)}$ via the distribution function $g(p)$. The off-diagonal slip coefficients are related to each other via Onsager relations.

**Results**

The coefficients $\alpha_{ik}$ are expresed via the integrals:

$$\left\{\begin{array}{c} L_n^{(\alpha)}(z) \\ K_n^{(\alpha)}(z) \\ M_n^{(\alpha)}(z) \\ Q_n^{(\alpha)}(z) \\ V_n^{(\alpha)}(z) \\ G_n^{(\alpha)}(z) \\ S_n^{(\alpha)}(z) \end{array}\right\} = -\int_{(v_z>0)} \frac{d^3p}{(2\pi\hbar)^3} \frac{\partial f_\beta^0}{\partial \varepsilon} \tau_{\alpha\gamma}^{n-1} e^{-\tau_{\gamma\beta}^{-1} z/v_z} \left\{\begin{array}{c} p_x^2 v_z^n \\ p_x^2 v_z^{n-1} \\ p_x^2 v_z^{n-1} v^2 \\ p_x v_x v_z^{n-2} \\ p_x v_x v_z^{n-2} v^2 \\ \varepsilon_p p_x v_x v_z^{n-2} \\ \varepsilon_p p_x v_x v_z^{n-2} v^2 \end{array}\right\}$$

and have the following bounds:

$$\frac{\sum_\alpha L_2^{(\alpha)}(0)}{\sum_\alpha L_1^{(\alpha)}(0)} < \alpha_{11} < \frac{\sum_\alpha L_3^{(\alpha)}(0)}{\sum_\alpha L_2^{(\alpha)}(0)}$$

$$\alpha_{12} > 0; \qquad \alpha_{21} < -(K_2^+(0) - K_2^-(0))\left[\frac{\sum_\alpha L_2^{(\alpha)}(0)}{\sum_\alpha L_1^{(\alpha)}(0)} - \frac{K_3^+(0) - K_3^-(0)}{K_2^+(0) - K_2^-(0)}\right]$$

$$\alpha_{13} < 0; \qquad 2\alpha_{31} > -\alpha_{11}[M_2^+(0) + M_2^-(0)] + M_3^+(0) + M_3^-(0);$$

$$\alpha_{22} > A\sum_\alpha \frac{Q_3^{(\alpha)}(0)}{Q_2^{(\alpha)}(0)}; \qquad T\alpha_{33} < \sum_\alpha [S_3^{(\alpha)}(0) - \frac{V_3^{(\alpha)}(0) G_2^{(\alpha)}(0)}{Q_2^{(\alpha)}(0)} + B\frac{V_3^{(\alpha)}(0)}{Q_2^{(\alpha)}(0)}];$$

$$\alpha_{32} < A\left[\frac{V_3^+(0)}{Q_2^-(0)} - \frac{V_3^+(0)}{Q_2^-(0)}\right];$$

$$T\alpha_{23} < G_3^+(0) - G_3^-(0) - \frac{Q_3^+(0)G_2^+(0)}{Q_2^-(0)} + \frac{Q_3^+(0)G_2^-(0)}{Q_2^-(0)} + B\sum_\alpha \frac{Q_3^{(\alpha)}(0)}{Q_2^{(\alpha)}(0)}$$

$$A = m_+ m_- \frac{Q_2^+(0)Q_2^-(0)}{m_+ Q_2^+(0) + m_- Q_2^-(0)}; \qquad 16B = \frac{5T}{172}\tau_+\tau_- \frac{N_+ - N_-}{N_+\tau_+ + N_-\tau_-};$$

where $N_\pm$ and $\tau_\pm$ are the number of particles and the relaxation times for spin-up and spin-down components. The bounds for the slip coefficients in the limiting cases of Boltzmann and degenerate gases are given in Figs. 1-5.

Note that at low temperatures all heat-related coefficients $\alpha_{31}, \alpha_{32}, \alpha_{33}$ contain extra powers of $T/T_F$, and therefore are small and insignificant.



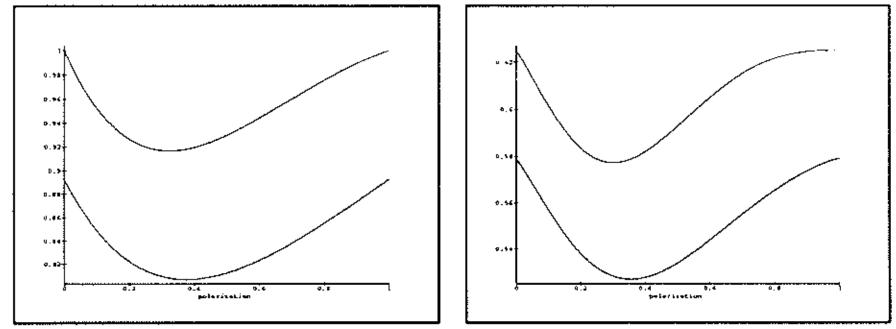

Figure 1: The coefficients $\frac{\alpha_{11}/\tau_+}{\sqrt{8T/m\pi}}$ in the Boltzmann, and $\frac{\alpha_{11}}{\tau_+ v_{F+}}$ in degenerate region.

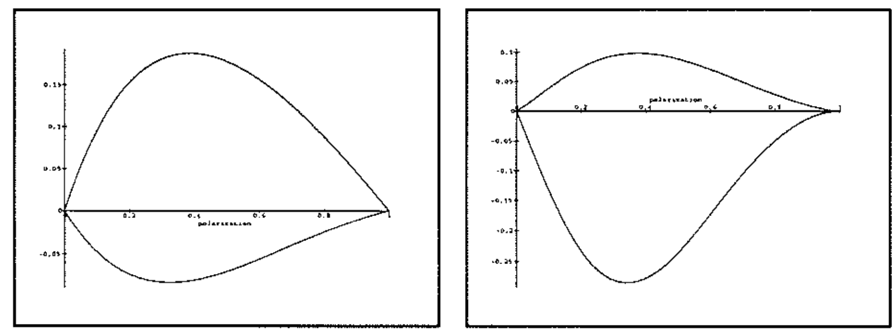

Figure 2: The coefficients $\frac{8\alpha_{21}}{TN_+\tau_+^2}$ in the Boltzmann, and $\frac{5\alpha_{21}}{mN_+\tau_+^2 v_{F+}^2}$ in degenerate region.

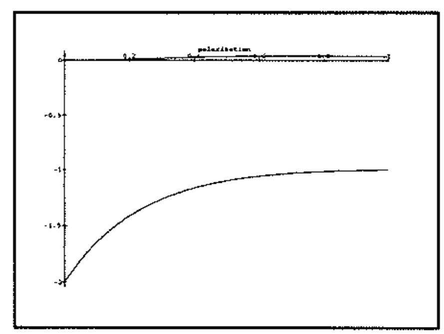

Figure 3: The coefficient $\frac{25m\alpha_{31}}{T^2 N_+\tau_+^2}$ in the Boltzmann region.

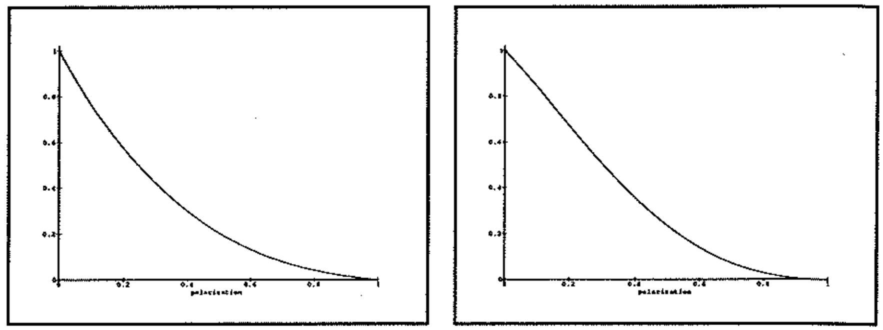

Figure 4: The coefficients $\frac{8\alpha_{22}}{\sqrt{\frac{2mT}{\pi}}N_+\tau_+^2}$ in the Boltzmann, and $\frac{16\alpha_{22}}{3mN_+\tau_+^2}$ in degenerate region.

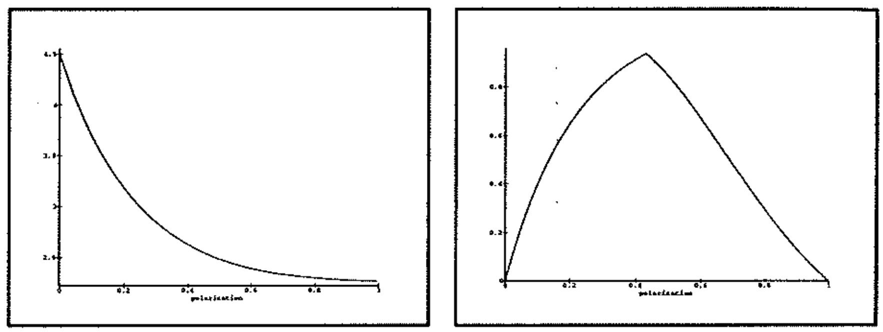

Figure 5: The coefficients $\frac{2m\alpha_{11}}{TN_+\tau_+^2}\sqrt{\frac{m\pi}{2T}}$, and $\frac{16\alpha_{12}}{TN_+\tau_+^2}\sqrt{\frac{m\pi}{2T}}$ in the Boltzmann region.

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