

# **Zero-Temperature Relaxation in Spin-Polarized Fermi Liquids**

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*We discuss the effect of zero-temperature attenuation, which has been recently observed in spin dynamics of spin-polarized Fermi liquids, on other Fermi-liquid processes. The transfer of this attenuation mechanism from transverse spin dynamics to longitudinal processes can be caused by the magnetic dipole interaction, namely, by the direct dipole processes and the dipole coupling between the transverse spin dynamics and the longitudinal transport and relaxation processes. We calculated the zero-temperature sound attenuation in spin-polarized Fermi liquids, corrections to the threshold of spin-wave (Castaing) instability, and the effective zero-temperature viscosity and longitudinal relaxation time in low- and high-frequency regimes.*

## **1. INTRODUCTION**

One of the recent developments in physics of spin-polarized Fermi liquids was a prediction and observation of peculiar zero-temperature attenuation in transverse spin dynamics. In contrast to all other dissipative processes in pure Fermi liquids, the transverse relaxation time  $\tau_{\perp}$  and the coefficient of transverse spin diffusion  $D_{\perp}$  do not increase with decreasing temperature as  $1/T^2$ , but saturate and remain finite even at  $T \rightarrow 0$ . By transverse dynamics we mean the dynamics of transverse components of magnetization perpendicular to its equilibrium direction; the transverse processes are excited by inhomogeneous tipping of spins in NMR experiments. Longitudinal processes, *i.e.*, processes which do not change the direction of polarization, do not exhibit any anomalous zero-temperature attenuation mechanisms in exchange systems irrespective of spin polarization.

The existence of the zero-temperature attenuation in transverse dynamics, which was predicted on the basis of general conservation law and symmetry arguments<sup>1</sup> (see also review<sup>2</sup>), was recently confirmed by

direct transport calculations for degenerate Fermi gases<sup>3-5</sup> and, later, dense Fermi liquids.<sup>6</sup> This temperature saturation of transverse diffusion and relaxation parameters has been observed in the low-temperature spin dynamics experiments in spin-polarized liquid  ${}^3\text{He}\uparrow$  and  ${}^3\text{He}\uparrow - {}^4\text{He}$  mixtures.<sup>8</sup> At present, the existence of this effect is well established.

The reason for such an unusual behavior is that the transverse relaxation and spin diffusion at low temperatures are determined by the collisionless decay of magnons. Spin polarization of the Fermi liquid opens the phase space between spin-up and spin-down Fermi spheres allowing the decay processes for magnons with finite  $\mathbf{k}$  (inhomogeneously tipped spins) even at  $T=0$ . Mathematically, this zero-temperature attenuation in transverse spin dynamics of Fermi liquids can be described by a pole contribution in the transverse component of the interaction function (molecular field), and is, in this sense, similar to the Landau damping in collisionless plasma.<sup>5</sup>

By the order of magnitude, the zero-temperature relaxation time in transverse channel is  $\tau_{\perp}(T=0) \sim (Nv_F\sigma)^{-1}(T_F/\beta H)^2$  for a system of fermions with the Fermi velocity (temperature)  $v_F$  ( $T_F$ ), magnetic moment  $\beta$ , effective cross-section  $\sigma$ , and the density  $N$ . Since the usual temperature-driven relaxation time is  $\tau_{\perp}(H=0) \sim (Nv_F\sigma)^{-1}(T_F/T)^2$ , the transition from the temperature-driven to polarization-driven attenuation in the transverse channel occurs at the temperature  $T_a \sim \beta H$  when the phase space between spin-up and spin-down Fermi spheres becomes comparable to the thermal smearing of the Fermi surfaces.

This dissipation channel is the only known zero-temperature relaxation mechanism in *pure* Fermi liquids for low-frequency long-wave processes. An obvious question is whether this dissipation mechanism is coupled to and affects other Fermi-liquid processes, including the longitudinal ones. This question will be addressed below.

In principle, there are three general mechanisms that couple longitudinal and transverse processes in homogeneous systems: the magnetic dipole-dipole interaction, the spin-lattice interaction in solid-state electron systems, and the non-linearity of equations of motion. [In inhomogeneous conditions with the varying direction of magnetization  $\mathbf{M}(\mathbf{r})$ , the difference between longitudinal and transverse processes becomes superficial, and all diffusion and relaxation processes should be determined by the shortest relaxation time, namely,  $\tau_{\perp}$ . Such a situation can occur, for example, in an inhomogeneous magnetic field when<sup>9</sup> the stationary solution for magnetization  $\mathbf{M}$  has domain structure, often with a thick domain wall. Within this wall, the direction of  $\mathbf{M}(\mathbf{r})$  gradually changes by  $180^\circ$ ].

In what follows, we will study the dipole coupling in helium systems, which is quite noticeable at high polarization.<sup>10</sup> We will not consider here

the non-linear coupling though this coupling could also lead to interesting effects especially close to or as a result of the spin-wave instability in an inhomogeneous setting (the Castaing instability). The spin-lattice interaction in electron systems is, to a large extent similar to dipole interaction in helium; however, the zero-temperature transverse attenuation in metals has certain features that require a separate analysis.<sup>11</sup>

In systems such as liquid  ${}^3\text{He}\uparrow$  and  ${}^3\text{He}\uparrow - {}^4\text{He}$  mixtures, (nuclear) magnetic dipole-dipole interaction is weak, and the effect of transverse attenuation on longitudinal dynamics can be observed only at ultra-low temperatures when the temperature-driven longitudinal attenuation vanishes. For liquid  ${}^3\text{He}\uparrow$  this corresponds to the temperatures below the superfluid transition when the theory of normal Fermi liquids cannot be applied directly. Therefore, the results can be applied, without modifications, mostly to liquid  ${}^3\text{He}\uparrow - {}^4\text{He}$  mixtures.

In  ${}^3\text{He} - {}^4\text{He}$  mixtures with  $F_0^{(s)} < 0$ , the zero sound cannot propagate. The longitudinal sound-like oscillations in mixtures are<sup>12</sup> the high-frequency first sound (sound oscillations of the superfluid  ${}^4\text{He}$  renormalized by the  ${}^3\text{He}$  impurities), Landau spin zero sound in the  ${}^3\text{He}$  component (in polarized mixtures, the longitudinal spin oscillations are coupled to the density oscillations), and the second sound which is, at  $T=0$ , the density oscillation mode in the  ${}^3\text{He}$  component of the mixture. The most interesting modes are the zero sound in  ${}^3\text{He}$ , second sound in  ${}^3\text{He} - {}^4\text{He}$ , and the high-frequency first sound. In all these cases, the form of dipolar coupling between the longitudinal and transverse modes are similar, though the full equations involving different longitudinal modes are somewhat different. Nevertheless, the attenuation of all these sound modes can be described in terms of the effective relaxation time,  $\tau_{\text{eff}}$ , and the effective Fermi-liquid viscosity,  $\eta_{\text{eff}} = \rho v_F^2 \tau_{\text{eff}} (1 + F_1^{(s)}/3)/5$ , via usual hydrodynamic and/or transport equations. The values  $\tau_{\text{eff}}$  and  $\eta_{\text{eff}}$  of the effective relaxation time and viscosity should be mode-independent. For this reason, we prefer to calculate the mode-independent effective relaxation parameters rather than the attenuation of each particular mode.

We will start from a calculation of the zero-temperature attenuation for the zero-sound in a polarized generic Fermi liquid. The sound attenuation will allow us to extract the effective mode-independent zero-temperature longitudinal relaxation time  $\tau_{\text{eff}}$  and the effective Fermi-liquid viscosity  $\eta_{\text{eff}}$ . These expressions, in turn, can be substituted into the usual hydrodynamic and transport equations for  ${}^3\text{He}$  and  ${}^3\text{He} - {}^4\text{He}$  thus letting us to avoid separate calculations for different helium systems and modes.

We will also look at dipole corrections for the threshold of Castaing instability. Though the effective zero-temperature longitudinal relaxation parameters are quite small because of the weakness of dipole interaction,

these parameters provide the real zero-temperature cut-offs for longitudinal relaxation and transport. Since liquid helium, in contrast to electron systems, does not have any impurities, one may expect to observe these limiting cut-offs at ultra-low temperatures in highly polarized  ${}^3\text{He}\uparrow$  or  ${}^3\text{He}\uparrow - {}^4\text{He}$  mixtures.

## 2. SOUND ATTENUATION IN SPIN-POLARIZED FERMION LIQUIDS AT $T=0$

In general, the transverse zero-temperature attenuation in spin-polarized Fermi liquids exists because the shift between the Fermi spheres for spin-up and spin-down particles opens the phase space for attenuation of quasiparticles in spin-flip processes. The corresponding collision integral contains the integrand of the type<sup>3,6</sup>

$$d^3p_1 d^3p_2 d^3p_3 d^3p_4 \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 - \hbar\omega) \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \\ \times W[n_{1\uparrow}n_{2\uparrow}(1-n_{3\uparrow})(1-n_{4\downarrow}) + n_{1\uparrow}n_{2\downarrow}(1-n_{3\downarrow})(1-n_{4\downarrow})] \quad (1)$$

and goes to zero at zero polarization ( $W$  is the probability of the corresponding process). The processes of the type (1) involve the spin flip. Therefore, within the exchange approximation, such a collision integral can appear only for the transverse processes, i.e., for the off-diagonal component  $n_{\uparrow\downarrow}$  of the single-particle density matrix  $\hat{n}$ .

Without the dipole interaction, the collision integrals  $iL_{\uparrow,\downarrow}$  for longitudinal processes involving spin-up and spin-down (quasi-)particles vanish at  $T=0$ , while the longitudinal processes are decoupled from the transverse ones. Then the kinetic equations for spin-up and spin-down (quasi-)particles at  $T=0$  have the usual form,

$$0 = \delta n_{\uparrow,\downarrow}(\omega - \mathbf{k} \cdot \mathbf{v}) + \mathbf{k} \cdot \mathbf{v} \frac{\partial n_{\uparrow,\downarrow}}{\partial \varepsilon_{\uparrow,\downarrow}} \\ \times \int [f^{(s)}(\mathbf{p}, \mathbf{p}')(\delta n_{\uparrow}(\mathbf{p}') + \delta n_{\downarrow}(\mathbf{p}')) \pm f^{(a)}(\mathbf{p}, \mathbf{p}')(\delta n_{\uparrow}(\mathbf{p}') - \delta n_{\downarrow}(\mathbf{p}'))] d\Gamma' \quad (2)$$

where  $f^{(s,a)}$  are the symmetric and anti symmetric parts of the exchange interaction function, and  $\delta n_{\uparrow,\downarrow}$  are the small deviations of the distribution functions for up and down spins from the equilibrium. The dipole interaction changes this equation in two ways. First, it adds to the l.h.s. of Eq. (2) the dipole collision integrals  $iL_{\uparrow,\downarrow}$  with the structure (1), which do not vanish at  $T=0$  and lead directly to the zero-temperature attenuation for longitudinal processes. Second, it changes the interaction function  $f$  (the

effective molecular field) in the r.h.s. in such a way that the integral term involves coupling to transverse (off-diagonal) components of the single-particle density matrix  $\delta n_{\uparrow\downarrow}$ , and, therefore, transfers the zero-temperature transverse attenuation from the transverse into the longitudinal channel.

The former, “direct” dipole processes lead to the direct spin-flip involving a dipole collision integral with the integrand (1) and the dipole vertex  $W$ . In the latter, “indirect” processes, the dipole part of the effective molecular field  $f$  couples longitudinal and transverse processes (equations); then the integral (1) appears in the transverse equation with the transverse *exchange* vertex  $W$ , and affects the longitudinal processes via the dipole interaction term in the coupling constant.

The dipole vertex  $W$ , which describes the direct dipole attenuation processes (1), differs from the exchange vertex by a small factor of the order of  $(E_d/T_F)^2$  where the characteristic dipole energy is  $E_d = \beta^2 Z^2 m^{3/2} T_F^{3/2} / h^3$ , and  $Z$  is the microscopic parameter which describes the difference between the (pole terms for) Fermi liquids and gases. The same factor describes the difference between the effective zero-temperature attenuation in the longitudinal channel  $\tau_{\text{eff}}(T=0)$ , which arises as a result of the direct dipole processes, and the exchange zero-temperature transverse attenuation  $\tau_{\perp}(T=0)$ .

The second, indirect mechanism of sound attenuation at  $T=0$ , is a result of magnetic dipole coupling between sound oscillations and transverse spin dynamics. This coupling transfers the zero-temperature attenuation from the transverse spin waves to the longitudinal sound, and the factor  $(E_d/T_F)^2$  appears in the coupling constant rather than in the transition probability  $W$ .

Since both—direct and indirect—dipole attenuation processes are weak, these processes can be studied separately, independently from each other. As a result of dipole processes, the transition from the temperature-driven to the polarization-driven zero-temperature *sound* attenuation should occur for the longitudinal sound in sub- $\mu\text{K}$  region, *i.e.*, at considerably lower temperature than the recently observed anisotropy temperature  $T_a$  at which the transverse attenuation loses its  $1/T^2$  dependence. Our results for sound attenuation will allow us to calculate the effective dipole zero-temperature contribution to viscosity  $\eta_{\text{eff}}(T=0)$ .

### A. Direct Processes

In this Section we will consider the direct dipole attenuation processes in the longitudinal channel and neglect the dipole coupling to transverse dynamics. Then the kinetic equations for spin-up and spin-down particles (2) should include the dipole collision integrals  $iL_{\uparrow,\downarrow}$  in the l.h.s. At  $T=0$ ,

exchange collisions in Fermi liquids do not result in attenuation for longitudinal processes, while the non-zero contributions to the *dipole* collision integrals  $L_{\uparrow,\downarrow}$  come from the following spin components

$$L_{(\uparrow,\downarrow)} = L_{(\uparrow,\downarrow)\uparrow,\uparrow\downarrow} + L_{(\uparrow,\downarrow)\uparrow,\downarrow\uparrow} + L_{(\uparrow,\downarrow)\downarrow,\uparrow\uparrow} + L_{(\uparrow,\downarrow)\downarrow,\downarrow\downarrow} \quad (3)$$

defined as

$$L_{(ij),kl} = \frac{1}{\hbar} \int |W|^2 \delta(\varepsilon_i + \varepsilon_{2j} - \varepsilon_{3k} - \varepsilon_{4l}) \delta(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \\ \times (n_i n_{2j} (1 - n_{3k})(1 - n_{4l}) - (1 - n_i)(1 - n_{2j}) n_{3k} n_{4l}) \frac{d^3 p_2 d^3 p_3 d^3 p_4}{(2\pi\hbar)^6} \quad (4)$$

( $i, j, k, l$  stand for  $\uparrow$  or  $\downarrow$ ). The probability  $W$  of the dipole process in Eq. (4) is equal to

$$|W|^2 = |\Gamma_{kl,ij}(\mathbf{p}_3, \mathbf{p}_4; \mathbf{p}, \mathbf{p}_2)|^2 \\ = (4\pi\beta^2 Z^2)^2 \left| \frac{(\mathbf{p} - \mathbf{p}_3)_z (\mathbf{p} - \mathbf{p}_3)_+}{(\mathbf{p} - \mathbf{p}_3)^2} - \frac{(\mathbf{p} - \mathbf{p}_4)_z (\mathbf{p} - \mathbf{p}_4)_+}{(\mathbf{p} - \mathbf{p}_4)^2} \right|^2 \quad (5)$$

and it is the same for all  $L_{ij,kl}$ . The vertex function  $\Gamma_{kl,ij}$  in (5) is expressed via the matrix elements of the dipole interaction  $U_{\sigma\sigma'}(\mathbf{q})$  as

$$\Gamma_{kl,ij}(\mathbf{p}_3, \mathbf{p}_4; \mathbf{p}, \mathbf{p}_2) = Z^2 (U_{kl,ij}(\mathbf{p} - \mathbf{p}_3) - U_{lk,ij}(\mathbf{p} - \mathbf{p}_4)) \\ U_{\sigma\sigma'}(\mathbf{q}) = \frac{4}{3} \pi\beta^2 \left[ \frac{3(\boldsymbol{\sigma} \cdot \mathbf{q})(\boldsymbol{\sigma}' \cdot \mathbf{q})}{q^2} - (\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}') \right] \quad (6)$$

where  $Z$  is the Fermi-liquid renormalization coefficient in the pole part of the single-particle Green's function. The equilibrium distribution makes the collision integral equal to zero,  $L_{ij,kl}\{n_{\uparrow,\downarrow}^{(0)}(\varepsilon)\} = 0$ .

The distribution function in the kinetic Eq. (2) has the form

$$n_{\uparrow,\downarrow}(\varepsilon) = n_{\uparrow,\downarrow}^{(0)}(\varepsilon_0) + \delta n_{\uparrow,\downarrow} = n_{\uparrow,\downarrow}^{(0)}(\varepsilon) + \Delta n_{\uparrow,\downarrow} \quad (7)$$

where

$$\Delta n_{\uparrow,\downarrow} = - \frac{\partial n_{\uparrow,\downarrow}^{(0)}}{\partial \varepsilon_{\uparrow,\downarrow}^{(0)}} \delta \varepsilon_{\uparrow,\downarrow} + \delta n_{\uparrow,\downarrow}$$

We will look for the eigenvalue of the kinetic equation using the perturbation expansion in the dipole interaction,  $\omega = \omega^{(0)} + \omega^{(1)} + \dots$ , where  $\omega^{(0)}$  is the sound eigenvalue,

$$\omega^{(0)} = sv_F k \quad (8)$$

in the absence of dipole interaction. As it is common to this type of kinetic Fermi-liquid problems, the equations can be simplified by the substitution

$$\Psi_{\uparrow,\downarrow} = \frac{\Delta n_{\uparrow,\downarrow}}{n_{\uparrow,\downarrow}^{(0)}(1 - n_{\uparrow,\downarrow}^{(0)})} \simeq -\omega^{(0)} \delta n_{\uparrow,\downarrow} / \beta_1 H(\mathbf{k} \cdot \mathbf{v}) \frac{\partial n_{\uparrow,\downarrow}^{(0)}}{\partial \varepsilon_{\uparrow,\downarrow}^{(0)}} \quad (9)$$

where we assumed that the external magnetic field is weak and introduced the usual Fermi-liquid renormalization of the particle magnetic moment,

$$\beta_1 = \beta / (1 + F_0^{(a)})$$

( $F_0^{(a)}$  is the zeroth harmonic of the antisymmetric Landau function).

Without the dipole interaction, the solution has the form

$$\delta n_{\uparrow,\downarrow} = -\frac{\mathbf{k}\mathbf{v}}{\omega^{(0)} - \mathbf{k}\mathbf{v}} \frac{\partial n_{\uparrow,\downarrow}^{(0)}}{\partial \varepsilon_{\uparrow,\downarrow}^{(0)}}$$

with

$$\Psi_{\uparrow,\downarrow} = \Psi = \frac{1}{\beta_1 H} \frac{\omega^{(0)}}{\omega^{(0)} - \mathbf{k}\mathbf{v}} \quad (10)$$

Here we assumed, as it is often done when studying the sound propagation in Fermi liquids, that the zeroth Landau harmonics,  $F_0^{(s,a)}$ , dominate the interaction function, and other harmonics can be neglected. The results for more general situations are much more cumbersome. In contrast to standard sound calculations, we prefer to use this approximation in order to simplify the dipole coupling terms rather than to simplify the integral Eq. (2). In this approximation, the eigenvalues in (8) is

$$\frac{s}{2} \ln \frac{s+1}{s-1} - 1 = \frac{1}{F_0^{(s)}} \quad (11)$$

Without the dipole interaction, the collision integral in (2) is zero. To get the main term in the collision integral (3)–(5), we should substitute

the solution of the kinetic equation without the dipole interaction into (4):

$$L_{ii,kl} = \frac{1}{h} \int |W|^2 \frac{d^3 \mathbf{p}_2 d^3 \mathbf{p}_3 d^3 \mathbf{p}_4}{(2\pi h)^6} \delta(\varepsilon_i + \varepsilon_{2j} - \varepsilon_{3k} - \varepsilon_{4l}) \delta(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \\ \times (\Psi_i + \Psi_2 - \Psi_3 - \Psi_4) n_i^0 n_{2j}^0 (1 - n_{3k}^0) (1 - n_{4l}^0), \quad \Psi_i = \Psi(\mathbf{p}_i) \quad (12)$$

Then the sound attenuation, i.e., the imaginary part of the eigenvalue  $\omega$  in the high frequency limit

$$\text{Re } \omega \gg 1/\tau_{\text{eff}} \quad (13)$$

becomes equal to

$$\text{Im } \omega = \int \frac{d^3 p}{(2\pi h)^3} \frac{\delta n}{\mathbf{k} \mathbf{v} \delta_+} L_+ \Big/ \int \frac{d^3 p}{(2\pi h)^3} \frac{(\delta n)^2}{\mathbf{k} \mathbf{v} \delta_+} \quad (14)$$

where we took into account that the sound waves are the joint oscillations of spin-up and spin-down particles,

$$L_+ = L_\uparrow + L_\downarrow, \quad n = n_\uparrow + n_\downarrow, \quad m = n_\uparrow - n_\downarrow, \quad \delta_+ = \delta_\uparrow + \delta_\downarrow, \quad \delta_{\uparrow, \downarrow} = \delta(\varepsilon - \varepsilon_{F, \uparrow, \downarrow}) \quad (15)$$

(the  $\delta$ -functions in denominators (14) are cancelled by the  $\delta$ -functions in  $\delta n$  (9)).

After straightforward algebra, Eq. (14) reduces to

$$\text{Im } \omega = \frac{\beta_1 H}{h\omega} \left[ \int \frac{d^3 p}{(2\pi h)^3} \frac{\mathbf{k} \mathbf{v} (\delta_\uparrow + \delta_\downarrow)}{(\omega - \mathbf{k} \mathbf{v})^2} \right]^{-1} \int |W|^2 \frac{d^3 p d^3 p_2 d^3 p_3 d^3 p_4}{(2\pi h)^9} \\ \times \delta(\varepsilon + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 - 2\beta_1 H) \delta(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \\ \times (\Psi + \Psi_2 - \Psi_3 - \Psi_4)^2 (n_\uparrow^0 n_{2\uparrow}^0 (1 - n_{3\uparrow}^0) (1 - n_{4\downarrow}^0) \\ + n_\uparrow^0 n_{2\downarrow}^0 (1 - n_{3\downarrow}^0) (1 - n_{4\uparrow}^0)) \quad (16)$$

The calculation of the integrals of the type (16), which appear in similar problems,<sup>3,6</sup> is rather complicated; we will not give the details. We used the method similar to that in the fourth Ref. 6. The substitution  $\mathbf{q} = \mathbf{p} - (\mathbf{p}_3 + \mathbf{p}_4)/2$  trivializes the integration over the energy  $\delta$ -function, especially in low magnetic field when one can disregard the term  $2\beta_1 H$  in its argument. After the integration over the  $\delta$ -functions, one can simplify the integral (16) to the form

$$\text{Im } \omega = \frac{E_d^2}{16\pi^5 h T_F} \left( \frac{\beta_1 H}{T_F} \right)^2 I(s, \cos \theta) \quad (17)$$

where  $\theta$  is the angle between the wave vector  $\mathbf{k}$  and the magnetic field  $\mathbf{H}$  (the  $z$ -axis), and

$$\begin{aligned}
 & I(s, \cos \theta) \\
 &= \left[ \frac{2s^2}{s^2 - 1} - s \ln \frac{s+1}{s-1} \right]^{-1} \left( \frac{T_F}{\beta_1 H} \right)^3 \int d^3 p_3 d^3 p_4 \sin \theta_q d\theta_q d\varphi_q |\mathbf{p}_4 - \mathbf{p}_3| \\
 & \quad \times \left| \frac{(\mathbf{q} + (\mathbf{p}_4 - \mathbf{p}_3)/2)_z (\mathbf{q} + (\mathbf{p}_4 - \mathbf{p}_3)/2)_+}{(\mathbf{q} + (\mathbf{p}_4 - \mathbf{p}_3)/2)^2} \right. \\
 & \quad \left. - \frac{(\mathbf{q} - (\mathbf{p}_4 - \mathbf{p}_3)/2)_z (\mathbf{q} - (\mathbf{p}_4 - \mathbf{p}_3)/2)_+}{(\mathbf{q} - (\mathbf{p}_4 - \mathbf{p}_3)/2)^2} \right|^2 \\
 & \quad \times \left( \frac{s |\mathbf{k}|}{s |\mathbf{k}| - \mathbf{k}(\mathbf{q} + (\mathbf{p}_3 + \mathbf{p}_4)/2)} + \frac{s |\mathbf{k}|}{s |\mathbf{k}| + \mathbf{k}(\mathbf{q} - (\mathbf{p}_3 + \mathbf{p}_4)/2)} \right. \\
 & \quad \left. - \frac{s |\mathbf{k}|}{s |\mathbf{k}| - \mathbf{k}\mathbf{p}_3} - \frac{s |\mathbf{k}|}{s |\mathbf{k}| - \mathbf{k}\mathbf{p}_4} \right)^2 \times n_{\uparrow}(\mathbf{q} + (\mathbf{p}_3 + \mathbf{p}_4)/2)(1 - n_{\downarrow}(\mathbf{p}_4)) \\
 & \quad \times (n_{\uparrow}(\mathbf{q} - (\mathbf{p}_3 + \mathbf{p}_4)/2)(1 - n_{\downarrow}(\mathbf{p}_3)) + n_{\downarrow}(\mathbf{q} - (\mathbf{p}_3 + \mathbf{p}_4)/2)(1 - n_{\downarrow}(\mathbf{p}_3)))
 \end{aligned} \tag{18}$$

where  $\mathbf{p}_{3,4}$  are dimensionless variables,  $\mathbf{p}_i \rightarrow \mathbf{p}_i/p_F$ ,  $|\mathbf{q}| = |\mathbf{p}_4 - \mathbf{p}_3|/2$  and  $n_{\uparrow, \downarrow}(\mathbf{x}) = 1 - \Theta(|\mathbf{x}| - 1 \mp \beta_1 H/2T_F)$ . The function  $I(\cos \theta)$  is plotted in Fig. 1 for several values of  $s$ .

These results define the direct dipole zero-temperature sound attenuation in polarized Fermi liquids. The corresponding effective value of the longitudinal relaxation time will be given in the next Section.

## B. Coupling to Transverse Spin Waves

In this sub-Section we will analyze the second dipole source of the zero-temperature attenuation for the longitudinal sound, namely the coupling between longitudinal and transverse processes. The main coupling mechanism, important in this context, is the dipole correction to the Landau interaction function which is, roughly speaking, the dipole component of the effective molecular field in spin-polarized systems. We will take into account only those spin components of the (weak) dipole interaction function which are responsible for the transfer of transverse attenuation into the longitudinal channels, and will disregard all other dipole contributions. Since the dipole forces, responsible for the coupling of transverse and longitudinal channels are weak, this coupling can be considered perturbatively in the lowest order.

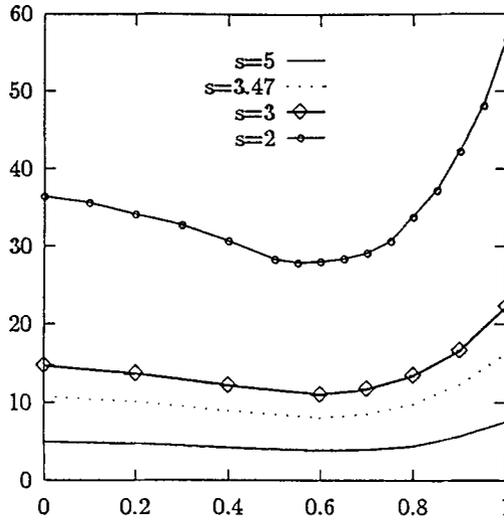


Fig. 1.  $I(s, x)$  as a function of  $x = \cos \theta$ , Eq. (16), for four values of  $s$ ,  $s = 2; 3; 3.47; 5$ .

The effect is simple. Without the dipole interaction, the low-frequency long-wave excitations in the Fermi system at  $T=0$  are non-attenuating density oscillations with linear spectrum, which are described by Eqs. (2), and attenuating transverse spin waves with quadratic spectrum, which are described by the equation<sup>2</sup>

$$\delta n_{\uparrow}(\omega - \mathbf{k} \cdot \mathbf{v} - \Omega) - \left[ \mathbf{k} \cdot \mathbf{v} \delta_+ + \frac{2}{\hbar} m^{(0)} \right] \int f^{(\omega)}(\mathbf{p}, \mathbf{p}') \delta n_{\uparrow}(\mathbf{p}') d\Gamma' = i\mathcal{L}_{\uparrow} \delta n_{\uparrow} \quad (19)$$

where  $\delta_+$  and  $m$  are defined in (10),  $\mathcal{L}_{\uparrow}$  is the appropriate spin component of the transverse exchange collision operator, and  $\Omega = \Omega_0 - \Omega_i = 2\beta_1 H/\hbar$  differs from the bare Larmor frequency  $\Omega_0 = 2\beta H/\hbar$  by the standard Fermi-liquid renormalization factor  $\beta_1 = \beta/(1 + F_0^{(\omega)})$ . Equation for the conjugate spin component of the density matrix,  $\delta n_{\downarrow}$ , differs from Eq. (19) by  $H \rightarrow -H$ .

In spin-polarized Fermi liquids, the transverse collision integral  $i\mathcal{L}_{\uparrow}$  has the structure (1) and is finite even at  $T=0$ <sup>1-8</sup> resulting in the zero-temperature attenuation of the spin waves. On the other hand, the density (sound) oscillations of  $\delta n$  are coupled to and accompanied by the oscillations of the longitudinal component of the magnetic moment  $\delta m$ , Eq. (2).

Magnetic dipole interaction couples the motion of longitudinal and transverse components of magnetization  $\delta m$  and  $\delta n_{\uparrow\downarrow}$  thus transferring the zero-temperature attenuation from transverse to longitudinal spin oscillations, and leading, in turn, to a finite sound attenuation (oscillations of  $\delta n$ ).

For simplicity, in this Section we will assume that the polarization is low. This will not only simplify the longitudinal equations, but will allow us to write transverse equations in a standard Fermi-liquid form without any microscopic complications inherent to the transverse equations in dense highly polarized Fermi liquids.<sup>6</sup> As in the previous Section, we will assume that the zeroth Landau harmonics,  $F_0^{(s,a)}$ , dominate the interaction function, and other harmonics can be neglected. We prefer to use this approximation in order to simplify the dipole coupling terms rather than to simplify the integral Eq. (2), (19).

In contrast to the previous Section, we will include the dipole interaction not into the collision integral, but into the Landau interaction function  $f$  responsible for coupling to transverse modes. Coupling of Eqs. (2) and (19) is provided by the magnetic dipole-dipole interaction with the Hamiltonian<sup>13,14</sup>  $U_{\sigma\sigma}(\mathbf{q})$ , Eq. (6). This term in the Hamiltonian leads to the dual change in the effective field  $f(\mathbf{p}, \mathbf{p}')$ . First, it is responsible for demagnetizing factors which, in an elliptical sample, are described by the demagnetizing field  $\mathbf{H}_d$ . The integration of dipolar interaction, necessary for the calculation of the demagnetizing field, is not trivial because of the divergence at small wave vectors. It is possible to show<sup>15-17</sup> that the demagnetizing field in spherical samples is, with good accuracy, equal to

$$\mathbf{H}_d = 4\pi(\mathbf{H}(\mathbf{M} \cdot \mathbf{H})/H^2 - \mathbf{M}/3) Z^2, \quad \mathbf{M} = (\beta/2) \text{Tr}_\sigma \int \sigma \hat{n}_\sigma d\Gamma \quad (20)$$

Note, that this equation for  $\mathbf{H}_d$  involves the exact value of the magnetization  $\mathbf{M}$ , and includes both the equilibrium and non-equilibrium terms  $\mathbf{M}_0$  and  $\delta\mathbf{M}$ .

What is more important in our context, the dipole interaction changes the effective interaction function. Here the divergencies in the dipole integrand do not cause any problems. The full antisymmetrized vertex function contains two diagrams which differ from each other by interchanged outgoing lines. The calculation of one of these diagrams and the limit  $\Gamma''$  of the corresponding vertex function yields the following addition to the Landau interaction function:

$$f_{\alpha\beta,\gamma\delta}(\mathbf{p}, \mathbf{p}') = f^{(s)}(\mathbf{p}, \mathbf{p}') \delta_{\alpha\gamma} \delta_{\beta\delta} + \boldsymbol{\sigma}_{\alpha\gamma} \cdot \boldsymbol{\sigma}_{\beta\delta} f^{(a)}(\mathbf{p}, \mathbf{p}') \\ + \frac{4}{3} \pi Z^2 \beta^2 \left[ \frac{3(\boldsymbol{\sigma}_{\beta\gamma} \cdot \mathbf{q})(\boldsymbol{\sigma}_{\alpha\delta} \cdot \mathbf{q})}{q^2} - (\boldsymbol{\sigma}_{\beta\gamma} \cdot \boldsymbol{\sigma}_{\alpha\delta}) \right] \quad (21)$$

(cf.  $\Gamma_{kl,ij}(\mathbf{p}_3, \mathbf{p}_4; \mathbf{p}, \mathbf{p}_2)$ , Eq. (6). Equation (21) contains only one of the diagrams for  $\Gamma^{(a)}$ . The other diagram is already included into the term with  $\delta\mathbf{M}$  in the demagnetizing field  $\mathbf{H}_d$  (20). The substitution of the dipole terms (20), (21) into the commutator in the equations of motion,

$$[\hat{n}, \hat{\varepsilon}], \varepsilon_{xy} = \varepsilon_{xy}^{(0)} - \beta \boldsymbol{\sigma}_{xy} \cdot \delta \mathbf{H}_d + \int f_{\alpha\beta, \gamma\delta}(\mathbf{p}, \mathbf{p}') \delta n'_{\delta\beta} d\Gamma' \quad (22)$$

leads to the appearance of new terms in Eqs. (2), (19).

Of all emerging small dipole terms, we are interested only in the terms that couple longitudinal and transverse equations. All other dipole terms, including the demagnetizing factors, lead to insignificant corrections to the spectra and are irrelevant in our context. We will neglect all these terms from the beginning. The coupling dipole terms change Eqs. (2) for oscillations of particle and spin density  $\delta n^+ \equiv \delta n$  and  $\delta n^- \equiv \delta m$  as

$$\begin{aligned} 0 = & \delta n^\pm (\omega - \mathbf{k} \cdot \mathbf{v}) - \mathbf{k} \cdot \mathbf{v} (\delta_\uparrow + \delta_\downarrow) \frac{\pi^2 \hbar^3}{p_F m} F_0^{(s,a)} \int \delta n'^\pm d\Gamma' \\ & - \mathbf{k} \cdot \mathbf{v} (\delta_\uparrow - \delta_\downarrow) \frac{\pi^2 \hbar^3}{p_F m} F_0^{(a,s)} \int \delta n'^\mp d\Gamma' \\ & - \frac{4}{3} \pi \beta^2 Z^2 \mathbf{k} \cdot \mathbf{v} \delta_\mp \int \left[ \delta n'_{\uparrow\uparrow} \frac{3q_- q_-}{q^2} + \delta n'_{\downarrow\downarrow} \frac{3q_+ q_+}{q^2} \right] d\Gamma' \quad (23) \end{aligned}$$

Similar dipolar coupling corrections appear in Eqs. (19) for  $\delta n_{\uparrow\uparrow}$  and  $\delta n_{\downarrow\downarrow}$ ,

$$\begin{aligned} i\hat{\mathcal{L}}_{\uparrow\uparrow, \uparrow\uparrow} = & \delta n_{\uparrow\uparrow, \uparrow\uparrow} (\omega - \mathbf{k} \cdot \mathbf{v} \mp \Omega) - \left[ \mathbf{k} \cdot \mathbf{v} \delta_\pm \pm \frac{2}{\hbar} m^{(0)} \right] \\ & \times \left( \frac{\pi^2 \hbar^3}{p_F m} F_0^{(a)} \int \delta n'_{\uparrow\uparrow, \uparrow\uparrow} d\Gamma' + \frac{4}{3} \pi \beta^2 Z^2 \int \delta n' - \frac{3q_\pm q_\pm}{q^2} d\Gamma' \right) \quad (24) \end{aligned}$$

We are interested in the sound attenuation caused by zero-temperature non-vanishing part of the collision operator  $\hat{\mathcal{L}}_{\uparrow\uparrow, \uparrow\uparrow}$ . In the absence of dipole coupling, the (zero-) sound solution of Eq. (23) has the form

$$\begin{aligned} \delta n^- = & \text{const} \times \frac{\mathbf{k} \cdot \mathbf{v}}{\omega - \mathbf{k} \cdot \mathbf{v}} \frac{\beta_1 H}{T_F} \frac{F_0^{(s)}}{F_0^{(s)} - F_0^{(a)}} \\ \delta n^+ = & \text{const} \times \frac{\mathbf{k} \cdot \mathbf{v}}{\omega - \mathbf{k} \cdot \mathbf{v}} \end{aligned} \quad (25)$$

while the spectrum is

$$\omega^{(0)} = uk = sv_F k, \quad \frac{1}{F_0^{(s)}} = \frac{s}{2} \ln \frac{s+1}{s-1} - 1 \quad (26)$$

(below we will consider *const* in Eq. (25) equal to 1). Since we are looking for a small imaginary correction to this spectrum associated with the collision operator  $\hat{L}_{\uparrow\uparrow, \uparrow\uparrow}$  in Eq. (24), we can neglect all dipole terms except for those leading to attenuation. The solution of Eqs. (24) can be expanded in spherical harmonics as

$$\begin{aligned} \delta n_{\uparrow\uparrow, \uparrow\uparrow} &= n_{\uparrow\uparrow, \uparrow\uparrow}^{(0)} Y_{0,0} + n_{\uparrow\uparrow, \uparrow\uparrow}^{(1)} Y_{1,0} + n_{\uparrow\uparrow, \uparrow\uparrow}^{(1,1)} Y_{1,1} + n_{\uparrow\uparrow, \uparrow\uparrow}^{(1,-1)} Y_{1,-1} \\ Y_{0,0} &= \frac{1}{\sqrt{4\pi}}, \quad Y_{1,0} = i \sqrt{\frac{3}{4\pi}} \cos \theta, \quad Y_{1,\pm 1} = \mp i \sqrt{\frac{3}{8\pi}} \sin \theta e^{\pm i\phi} \end{aligned} \quad (27)$$

If we are interested only in the coupling-induced sound attenuation, we need only the contribution to  $\delta n_{\uparrow\uparrow, \uparrow\uparrow}$  that is proportional to  $\mathbf{k}$ . These terms in the spherical harmonics are equal to

$$\begin{aligned} n_{\uparrow\uparrow, \uparrow\uparrow}^{(0)} &= \frac{\Lambda}{4\sqrt{3}} \frac{ihkv_F J_{\pm}^{(1,0)}}{\beta\beta_1 H^2} \frac{ih/\tau_{\perp}}{2\beta_1 H \mp ih/\tau_{\perp}} \\ \tilde{n}_{\uparrow\uparrow, \uparrow\uparrow}^{(1)} &= \mp \frac{\Lambda}{2\sqrt{3}} \frac{ihkv_F I_{\pm}^{(0,0)}}{\beta_1 H(2\beta_1 H \mp ih/\tau_{\perp})} \left[ 1 + \sqrt{4\pi} \frac{J_{\pm}^{(1,0)}}{I_{\pm}^{(0,0)}} \right] \\ n_{\uparrow\uparrow, \uparrow\uparrow}^{(1,\pm)} &= \mp \sqrt{\frac{\pi}{3}} \Lambda \frac{ihkv_F J_{\pm}^{(1,\pm 1)}}{\beta_1 H(2\beta_1 H \mp ih/\tau_{\perp})} \end{aligned} \quad (28)$$

where

$$\begin{aligned} \Lambda &= \frac{\sqrt{2} E_d (\beta_1 H)^2}{\pi^2 T_F} \frac{F_0^{(s)}}{T_F (F_0^{(s)} - F_0^{(a)})}, \quad E_d = \beta^2 Z^2 m^{3/2} T_F^{3/2} / \hbar^3 \\ I_{\pm}^{(\alpha,\beta)} &= \int I_{\pm}(\theta, \phi) Y_{\alpha,\beta}^* d\phi \sin \theta d\theta \\ J_{\pm}^{(\alpha,\beta)} &= \int I_{\pm}(\theta, \phi) Y_{1,0}^* Y_{\alpha,\beta}^* d\phi \sin \theta d\theta \\ I_{\pm}(\theta, \phi) &= \int \frac{\mathbf{k} \cdot \mathbf{v}}{\omega^{(0)} - \mathbf{k} \cdot \mathbf{v}} \frac{q_{\pm} q_z}{q^2} d\Omega' \end{aligned} \quad (29)$$

and the  $z$ -axis is chosen along the magnetic field.

This expression for  $\delta n_{\uparrow, \downarrow}$  should be substituted into Eqs. (23). The main imaginary contribution comes from the term

$$\frac{3q_z}{q^2} \text{Im}[\delta n'_{\uparrow} q_- + \delta n'_{\downarrow} q_+] = \frac{kv_F}{\tau_{\perp}} \Lambda_1 \gamma(\theta, \phi) \quad (30)$$

where

$$\begin{aligned} \gamma(\theta, \phi) = & \left[ \frac{iI_+^{(1,0)} Q_{0,0}}{\sqrt{3}(1+F_0^{(a)})} - \frac{i}{\sqrt{3}} (I_+^{(0,0)} + \sqrt{4\pi} J_+^{(1,0)}) Q_{1,0} \right. \\ & \left. - i \sqrt{\frac{4\pi}{3}} (J_+^{(1,1)} Q_{1,1} + J_+^{(1,-1)} Q_{1,-1}) \right] \end{aligned} \quad (31)$$

$$\Lambda_1 = \frac{1}{\sqrt{8}} \frac{E_d}{\pi^2 T_F} \Lambda \frac{h^2}{(\beta_1 H)^2}, \quad Q_{\alpha, \beta}(\theta, \phi) = \int Y_{\alpha, \beta}(\theta', \phi') \frac{q_- q_z}{q^2} d\Omega'$$

As a result, all the angular dependence of the coefficients in the remaining integral equations is given by the integrals  $Q$ .

Eq. (30) should be substituted into (23). After integration over energies, this equation reduces to

$$\begin{aligned} 0 = & (-s + \cos \theta) \delta n^{\pm} + F_0^{(s,a)} \cos \theta \int \delta n'^{\pm} \frac{d\Omega'}{4\pi} \\ & + \frac{\beta_1 H}{T_F} F_0^{(a,s)} \cos \theta \int \delta n'^{\mp} \frac{d\Omega'}{4\pi} \\ & + \frac{ikv_F}{2\tau_{\perp}} \Lambda_1 \gamma(\theta, \phi) \cos \theta \left( \left[ 1 + \frac{\beta_1 H}{T_F} \right] \pm \left[ 1 - \frac{\beta_1 H}{T_F} \right] \right) \end{aligned} \quad (32)$$

Without coupling, i.e., at  $\Lambda_1 = 0$ , the solution of this equation in the lowest order in polarization has the form (25). The perturbative coupling correction to this solution is

$$\begin{aligned} \text{Im } \omega = & \frac{(kv_F)^2 \beta_1 H}{2\tau_{\perp} T_F} \frac{\Lambda_1 F_0^{(s)}}{F_0^{(s)} - F_0^{(a)}} \Xi, \\ \Xi = & \int \frac{d\Omega'}{4\pi} \frac{\gamma(\theta, \phi) \cos \theta}{s - \cos \theta} \Big/ \int \frac{d\Omega'}{4\pi} \frac{\cos \theta}{(s - \cos \theta)^2} \end{aligned} \quad (33)$$

After calculating integrals, we get the following expression for  $\text{Im } \omega$ :

$$\text{Im } \omega = \frac{\hbar^2 (k v_F)^2}{32\pi^2 \tau_{\perp}} \left[ \frac{F_0^{(s)}}{F_0^{(s)} - F_0^{(a)}} \right]^2 \times \frac{E_d^2}{T_F^4} \left( \frac{k_{\perp}^2 k^2 - k_{\parallel}^2}{k^2} \Gamma_1(s) + \frac{4k_{\perp}^4 - 3k^2 k_{\parallel}^2 + k^4}{3k^4} \Gamma_2(s) \right) \quad (34)$$

where

$$\begin{aligned} \Gamma_1(s) &= 2s^2(s^2 - 1) \frac{w(s^2 - 3) - 1/3}{w(s^2 - 1) - 1} \left[ 2 \frac{w(3s^2 - 1) - 1}{1 + F_0^{(a)}} + s^2 - 3w(s^2 - 1)^2 - \frac{7}{5} \right] \\ \Gamma_2(s) &= 2s^2(s^2 - 1) \frac{w(s^2 + 3) - 1/3}{w(s^2 - 1) - 1} \left[ w(s^4 - 1) - \frac{s^2}{3} - \frac{1}{5} \right] \\ w(s) &= \frac{s}{2} \ln \frac{s+1}{s-1} - 1 \end{aligned} \quad (35)$$

and in the single-harmonic approximation  $w(s) = 1/F_0^{(s)}$ .

Note, that Eq. (34), in contrast to Eq. (17), does not contain the spin polarization explicitly. This seems surprising since in the absence of polarization the zero-temperature attenuation should vanish. However, the polarization enters (34) implicitly in two places. First,  $1/\tau_{\perp}$  is proportional to the square of polarization. Second, the calculation was performed and the equation is valid only in low-frequency conditions  $k v_F \ll \Omega_0$ . At higher frequencies the factor  $(k v_F)^2$  should be substituted by the square of the Larmor frequency  $\Omega_0$ . As a result, (34) contains polarization even to the higher power than Eq. (17). Therefore, the contribution of dipole coupling to the zero-sound attenuation is comparable to the direct dipole attenuation only at relatively high spin polarizations.

### 3. EFFECTIVE RELAXATION AND VISCOSITY

The above equations for sound attenuation in polarized generic Fermi liquid allow us to get the values of effective relaxation time and viscosity. Comparing Eqs. (17) and (34) with the standard expressions for (zero) sound attenuation in Fermi liquids, we immediately get the following equation for the effective relaxation time for direct

$$\begin{aligned} \frac{1}{\tau_{\text{eff}}} &= \frac{\text{Im } \omega}{\xi(s)} = \frac{E_d^2}{16\pi^5 \hbar T_F} \left( \frac{\beta_1 H}{T_F} \right)^2 \frac{I(s, \cos \theta)}{\xi(s)} \\ \xi(s) &= \frac{w^2(s^2 - 1)(3s^2 + 1) + 2w(s^2 - 1) - 1}{w(s^2 - 1) - 1} \end{aligned} \quad (36)$$

and indirect

$$\begin{aligned} \frac{1}{\tau_{\text{eff}}} &= \frac{\text{Im } \omega}{\xi(s)} \\ &= \frac{\hbar^2 (kv_F)^2}{32\pi^2 \tau_{\perp} \xi(s)} \left[ \frac{F_0^{(s)}}{F_0^{(s)} - F_0^{(\omega)}} \right]^2 \\ &\quad \times \frac{E_d^2}{T_F^4} \left( \frac{k_z^2 k^2 - k_z^2}{k^2} \Gamma_1(s) + \frac{4k_z^4 - 3k_z^2 k^2 + k^4}{3k^4} \Gamma_2(s) \right) \end{aligned} \quad (37)$$

dipole processes. The full zero-temperature relaxation time can be obtained from Eqs. (36) and (37) using the Matthiessen's rule.

As a result, we can introduce the effective field-driven viscosity at  $T=0$ ,

$$\eta_{\text{eff}} = \frac{1}{5} \rho v_F^2 \tau_{\text{eff}} (1 + F_1^{(s)}/3) \quad (38)$$

Note, that the viscosity depends on the angle between the velocity gradient ( $\mathbf{k}$ ) and the direction of polarization  $z$ . This anisotropy of the fluid dynamics in spin-polarized systems with dipole interaction is quite natural.

It is not surprising that effective relaxation times for direct and indirect processes are somewhat different. The most noticeable difference is the prefactor  $k^2$  in Eq. (37) which is absent in Eq. (36). At first glance, this could imply that Eq. (36) describes, in contrast to (37), the high frequency attenuation. However, this is not the case. Both relaxation times were calculated under the same condition  $\omega \gg 1/\tau_{\text{eff}}$  (by the Fermi liquid standards, this is a low-frequency regime for zero sound). The extra factor  $k^2$  in (37) is not the sign of some low-frequency attenuation mechanism, but reflects the fact that coupling between the longitudinal and transverse modes exists only in inhomogeneous conditions (the coupling factor  $\mathbf{k} \cdot \mathbf{v}$  in Eq. (23)). Since the calculation of (34) was performed for  $kv_F \ll \Omega_0$ , the factor  $(kv_F)^2$  at higher frequencies should be substituted by the Larmor frequency  $\Omega_0^2$  thus disappearing from the equations. Only at these frequencies the indirect attenuation becomes comparable, by the order of magnitude, to the direct dipole attenuation.

The high-frequency attenuation can be evaluated by the method similar to that used in the calculation of sound attenuation in Fermi liquids.<sup>18</sup> The non-vanishing at  $T=0$  part of the collision operator contains the integrand<sup>3,6</sup> of the type (1)

$$\begin{aligned} &d^3 p_1 d^3 p_2 d^3 p_3 d^3 p_4 \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 - 2\beta_1 H - \hbar\omega) \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \\ &\quad \times [n_{1\uparrow} n_{2\uparrow} (1 - n_{3\uparrow})(1 - n_{4\uparrow}) + n_{1\uparrow} n_{2\downarrow} (1 - n_{3\downarrow})(1 - n_{4\downarrow})] \end{aligned} \quad (39)$$

[For transparency of the results, we explicitly described the spin-flip term in the energy  $\delta$ -function. In the low-frequency calculations in Sec. 2, we disregarded  $\hbar\omega$  in the energy  $\delta$ -function]. This integral is similar to that studied in,<sup>3,6,18</sup> and depends, in a relatively simple way, on the relation between three energy parameters,  $\hbar\omega$ ,  $T$ , and  $\beta H$ . These parameters play similar roles in defining the region of the phase where the integrand is non-zero. The analysis shows that at zero  $\omega$  this integral for *dilute* Fermi systems can be written<sup>3,6</sup> as

$$\gamma_{\omega, T=0} \left( 1 + \left( \frac{2\pi T}{\Omega_0/2} \right)^2 \right)$$

where  $\gamma_{\omega, T=0}$  is the attenuation coefficient at zero temperature and frequency (the exact temperature dependence is a little bit more complicated<sup>4</sup>). For dense Fermi systems the frequency  $\Omega_0$  in this equation is renormalized.<sup>6</sup> Without the magnetic field, the integral (39) has the form<sup>18</sup>

$$\gamma_{\Omega_0, \omega=0} \left( 1 + \left( \frac{\omega}{2\pi T} \right)^2 \right)$$

irrespective of the density. We are interested in this integral at  $T=0$ , when it reduces to the form

$$\gamma_{T, \omega=0} \left( 1 + \left( \frac{\omega}{\Omega_0/2} \right)^2 \right) \quad (40)$$

(in dense Fermi liquids,  $\Omega_0 = 2\beta H$  in this equation should also be renormalized,  $\beta \rightarrow \beta_1$ ), where  $\gamma_{T, \omega=0}$  determines the usual low-frequency sound attenuation in Fermi liquids,

$$\gamma_{T, \omega=0} = \text{Im } k = \frac{\xi(s)}{s\tau_{\text{eff}}v_F} \quad (41)$$

#### 4. SPIN-WAVE INSTABILITIES

The non-linear coupling between longitudinal and transverse channels is enhanced close to the Castaing instability in transverse spin dynamics (see, e.g., Ref. 19 and references therein). We analyzed the dipole effects on the onset of Castaing instabilities in the presence of magnetization gradient  $\nabla M$ . Without the dipole effects, the instability occurs at  $k^2 = \mu k_i \nabla_i M$ , where  $\mu$  is the usual Leggett parameter (see below). One of the effects of the dipole interaction is that the instability condition becomes anisotropic

by acquiring terms of the form  $k_z^2$ ,  $\mu k_z \nabla_z M$ ,  $(\mu \nabla M)^2$ , and  $(\mu \nabla_z M)^2$ . However, these terms contain a small factor  $E_d/T_F$ , and we will not give here neither the lengthy calculations, nor the corresponding cumbersome expressions. What is more important, these anisotropic corrections do not have any fixed pre-determined signs so that it is impossible to say whether the onset of instabilities occurs earlier in certain directions.

Though this instability appears in transverse spin dynamics, one of its important features is that the Leggett parameter  $\mu = \Omega_i \tau / M$  in the equation for the onset of instability is proportional not to the transverse relaxation time  $\tau_\perp$ , but to the longitudinal time  $\tau_\parallel$ ,  $\mu = \Omega_i \tau_\parallel / M$ . [The use of the same notation  $\mu$  in the equation for the onset of instability,  $\mu = \Omega_i \tau_\parallel / M$ , and in the equation of spin dynamics,  $\mu = \Omega_i \tau_\perp / M$ , is somewhat confusing and is due to the fact that at the time when these equations were first derived, the difference between transverse and longitudinal diffusion coefficients was not known]. Since  $\tau_\parallel \propto 1/T^2$ , this means that the onset of instability  $k^2 = \mu k_i \nabla_i M$  happens, with decreasing temperature, at larger and larger wave vectors. The usual derivation of this instability condition assumes that the gradient of the longitudinal magnetization causes a large longitudinal diffusion current without any longitudinal oscillations, i.e., that  $1/\tau_\parallel \gg kv_F$ . These two conditions, taken together, limit the temperature range in which the instability can be observed to

$$T_F \gg T \gg T_F (\alpha a / L)^{1/4} / x^{1/3} \quad (42)$$

where  $\alpha$  is the degree of spin polarization,  $x$  in the molar density of the Fermi liquid, and  $L$  is the spatial scale of the polarization gradient (in  ${}^3\text{He} - {}^4\text{He}$  mixtures,  $x$  is the concentration of  ${}^3\text{He}$ ; in pure  ${}^3\text{He}$ ,  $x = 1$ ).

The dipole coupling between longitudinal and transverse channels leads to a substitution of  $\tau_\parallel$  by  $\tau_{\text{eff}}$  and modifies this restriction. At zero temperature, the instability occurs at  $k^2 = \mu_{\text{eff}} k_i \nabla_i M$ ,  $\mu_{\text{eff}} = \Omega_i \tau_{\text{eff}} / M$  under the condition  $1/\tau_{\text{eff}} \gg kv_F$ . The compatibility of these two equations requires high polarization with a small gradient,

$$E_d \gg T_F (a / \alpha^3 L)^{1/4}$$

As a result, the instability exists even at zero temperature though can be observed only for extremely small values of  $k$ .

## 5. CONCLUSIONS

We developed a theory of dipole coupling between longitudinal and transverse spin dynamics processes in spin-polarized Fermi liquids with an

emphasis on the transfer of zero-temperature transverse attenuation into the longitudinal channels. We calculated the zero-temperature dipole contribution to the sound attenuation in a generic Fermi liquid. This information allowed us to evaluate the effective mode-independent longitudinal relaxation time and viscosity at  $T=0$ . These coefficients determine the zero-temperature attenuation of different hydrodynamic and high-frequency modes in helium systems. The effect of the spin-lattice interaction in spin-polarized solid-state electron systems should be similar.

We determined the effect of dipole coupling on the spin-wave (Castaing) instability. As a result of dipole coupling, the Castaing instability does not disappear even at ultra-low temperatures, though its observation would require a relatively large experimental installation. In addition, the dipole interaction makes all the processes in spin-polarized Fermi liquids highly anisotropic.

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