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Mode Coupling in Quantized Quasi-$2D$ Systems

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Diffusion and localization in quantized quasi-$2D$ systems with random rough surfaces is discussed. The emphasis is on mode coupling and on comparison of channels with different types of surface correlators. For small size roughness, the shapes of the dependence of the diffusion coefficient D on the channel width L are universal. For large scale roughness, the mode coupling is often suppressed and $D(L)$ becomes very sensitive to the type of surface correlator. Experimentally, this is important for better quality channels and wave guides. The persistent sawtooth dependence $D(L)$ at large correlation length of roughness R, $R > L$, is a distinct signature of the power-law decay of the Fourier image of the surface correlator while the suppression of the sawtooth behavior points at the exponential decay. Thickness fluctuations with an exponential power spectrum lead to a new type of quantum size effect in transport which is responsible for large-scale oscillations of $D(L)$. In contrast to the usual sawtooth quantum size effect, these oscillations are not related to the quantization directly but result from the exponential opening of coupling for modes with small quantum numbers at certain values of L.

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1. INTRODUCTION

Rapid progress in system miniaturization leads to proliferation of ballistic and semi-ballistic systems. Often, the nanotechnological applications involve pure systems with large particle free paths for which quantization of motion becomes increasingly important. Particle scattering is responsible for coupling of these quantized modes. In miniature systems, one of the main mechanisms of particle scattering and, therefore, mode coupling is the scattering by surface inhomogeneities. As often, the helium systems present some of the most versatile testing grounds on which one can check experimentally various theoretical predictions. Below we consider the effect

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of scattering by random surface roughness on mode coupling, diffusion, and localization in quasi-$2D$ helium systems. The results describe the quantum size effect (QSE) in single-particle diffusion for quasiparticles with linear and quadratic spectra in systems with different types of surface roughness.

2. MODE COUPLING AND QUANTUM SIZE EFFECT

Helium systems provide a convenient tool for the study of the effect of wall scattering on transport[1]. In contrast to other liquids and solids, one can easily scan a wide range of particle mean free paths by simply changing the temperature or impurity concentration in ^{3}He$-$^{4}He mixtures. What is more, the formation of thin ^{4}He-rich layers near the walls can prevent energy and magnetic accommodation of ^{3}He quasiparticles on the walls. As a result, one gets experimental access to a unique, almost model system with locally specular scattering of ^{3}He quasiparticles with a practically quadratic energy spectrum. The results below are aimed mostly at ^{3}He quasiparticles and phonons in quantized quasi-$2D$ flow channels.

The QSE is caused by quantization of particle motion in the film, $p_x \rightarrow \pi n/L$ (below $\hbar = 1$), and splits the $3D$ energy spectrum $\epsilon(\mathbf{p})$ into a set of minibands or modes $\epsilon_n(\mathbf{q}) = \epsilon(\pi n/L, \mathbf{q})$ ($\mathbf{q}$ is the $2D$ momentum along the film). Experimentally, the linewidth of the quantum well modes $\epsilon_n(\mathbf{q})$ is often limited by the thickness fluctuations[2]. These fluctuations (or, in other words, surface roughness) are also responsible for mode coupling.

A standard picture of the QSE in transport is that the quantization leads to a sawtooth dependence of the transport coefficients on the channel thickness L irrespective of whether the scattering is due to bulk processes or wall collisions[3]. The positions of the saw teeth are universal, while their amplitude depends on the details of scattering. The collisions with inhomogeneous walls can result in both intraband scattering $\epsilon_n(\mathbf{q}) \rightarrow \epsilon_n(\mathbf{q}')$ and interband transitions $\epsilon_n(\mathbf{q}) \rightarrow \epsilon_{n'}(\mathbf{q}')$ (mode coupling). It is known that the amplitude of the saw teeth is determined almost exclusively by the mode coupling: if one freezes the interband transitions (coupling), the curve becomes almost smooth with insignificant kinks instead of the well-developed saw teeth.

When the characteristic lateral size (the correlation length) of surface inhomogeneities R is small in comparison to L, all allowed interband transitions are equally probable, and the QSE for the transport coefficients have the standard sawtooth shape. With increase in the size of inhomogeneities R, the mode coupling becomes exponentially suppressed, at least for the exponential surface correlators in the Fourier space, leading to a gradual disappearance of the sharp QSE signs. It has been assumed that in the case $R \gg L$ the QSE does not have any interesting features.

This assumption turned out to be wrong[4]. It turned out that the QSE in conductivity of metal films with large-scale thickness fluctuations $R \gg L$ acquires large smooth oscillations. Below we analyze single-particle diffusion and localization for quasiparticles with quadratic and linear spectra such as ^{3}He impurities and phonons in superfluid helium. Note that the exponent $\varphi(E)$ in the localization length $\mathcal{R}(E)$ in a quasi-2D system,

$$\mathcal{R}(E) = \mathcal{L}(E) \exp[\varphi(E)], \quad \varphi(E) = \pi m S(E) D(E),$$

and the quasiparticle mean free path $\mathcal{L}$ are determined by the diffusion coefficient D (here S is the number of minibands ϵ_n accessible for a particle with energy E).

3. MODE COUPLING FOR LARGE-SCALE ROUGHNESS

The explanation of the anomalous QSE at $R \gg L$ also involves the interband transitions[4]. In this case, most of the interband transitions, in contrast to the situation at $R \ll L$, are effectively suppressed. Some of these transition channels open one by one with increasing channel thickness giving rise to mode coupling for the modes with low quantum indices.

Scattering by surface inhomogeneities changes the lateral momentum by $\Delta q \sim \pi/R$. This is sufficient for the interband transition when the equation $\epsilon_n(\mathbf{q}) = \epsilon_{n'}(\mathbf{q} + \Delta\mathbf{q})$ has a solution with $n' \neq n$. Thus, the modes n and $n+1$ become coupled when $(\partial\epsilon_n/\partial q)\Delta q \sim \partial\epsilon(\pi n/L, q)/\partial n$. For both quadratic and linear spectra, this opening of the scattering-driven coupling of the modes n and $n+1$ occurs at the channel thickness

$$L_n/\Lambda \sim \sqrt{nR/\Lambda} \tag{1}$$

where $\Lambda = \pi/q$ is the quasiparticle wavelength. This mode coupling starts first for the lowest minibands, *i.e.*, for the grazing quasiparticles with the largest free paths. Therefore, the effect of mode coupling on the quasiparticle transport should be the strongest for small values of n.

The effect of the mode coupling can clearly be seen in Figures 1 (^{3}He impurities) and 2 (phonons) for the single-particle diffusion coefficients for quasiparticles with quadratic and linear spectra. The method of calculations is similar to the one described in Refs.4,5. Figure 1 presents D as a function of the dimensionless channel thickness L/Λ at four different values of R/Λ, $R/\Lambda = 0.1; 1; 10; 50$ with the corresponding marking of the curves. Figure 2 presents D as a function of the dimensionless phonon frequency, $\omega L/c$.

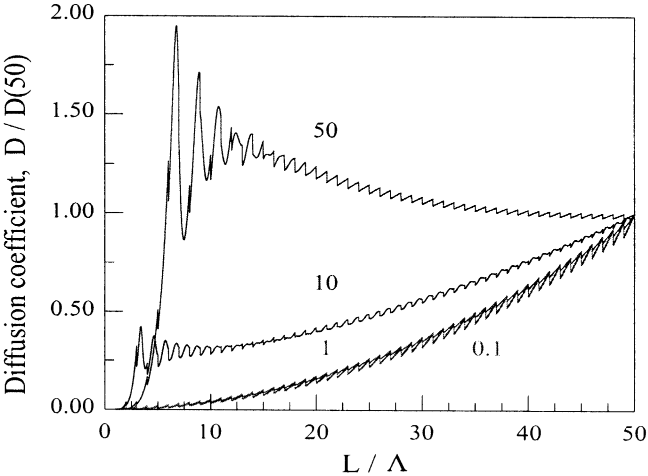

Fig. 1. Diffusion coefficient, Eq. (1), normalized to its value at $L/\Lambda = 50$, as a function of the channel thickness L at four different values of R/Λ. The saw-like dependence is typical for the standard quantum size effect.

These curves are plotted not at constant R/Λ as in Figure 1, but at constant $R/L = 0.1; 1; 10; 50$. Since the values of D at vastly different values of R/L (or R/Λ) differ from each other by several orders of magnitude, the sets of curves in each figure are normalized to the highest value of D, $D\,(50)$.

In both figures the curves for the two smaller values of R exhibit the standard sawtooth QSE while the curves with the two larger values of R exhibit large-scale peaks in the positions given by Eq. (1). These large smooth oscillations are manifestations of a new type of QSE. The positions of the peaks are determined not by the quantization conditions for momenta as for the standard QSE, but by the size of surface inhomogeneities or thickness fluctuations R. Since the curves in Figure 1 are plotted at constant R/Λ, the curves with $R/\Lambda = 10, 50$ display a gradual restoration of the standard sawtooth QSE at large values of L/Λ when R/L becomes small. This restoration does not occur in Figure 2 plotted at constant R/L; the curves with $R/L = 10, 50$ remain smooth with barely noticeable signs of the standard QSE.

If, instead of the Gaussian or exponential correlator in momentum space, one deals with a power-law correlator, the anomalous QSE cannot be observed even at large R/L and the curves always retain their sawtooth shape.

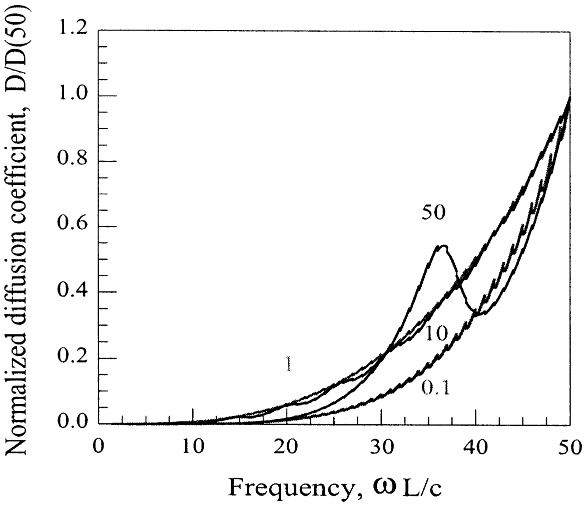

Fig. 2. Phonon diffusion coefficient, Eq. (1), normalized to its value at $\omega L/c = 50$, as a function of frequency ω at four different values of R/L. The saw-like dependence is typical for the standard quantum size effect.

In this case there is always a sufficient number of small-size inhomogeneities that ensure robust mode coupling at any value of R/L.

4. SUMMARY

In summary, diffusion and localization in quantized quasi-2D systems with random rough surfaces is discussed with an emphasis on mode coupling and comparison of channels and wave guides with various surface correlators. The results on quasiparticle diffusion and localization in quasi-2D helium channels are consistent with our earlier results on conductivity of metal films[4].

- At small R, the *shapes* of the curves for the dependence $D(L)$ are universal irrespective of the type of the surface correlator.
- At large R, the mode coupling is often suppressed and $D(L)$ becomes sensitive to the type of surface correlator.
- The persistent sawtooth dependence $D(L)$ at $R > L$ is a signature of

the power-law decay of the Fourier image of the correlator while the suppression of the sawtooth behavior points at the exponential decay.

- Thickness fluctuations with an exponential power spectrum lead to a new type of quantum size effect in diffusion and localization at large R which is responsible for large-scale oscillations of $D(L)$. These oscillations are not related to the quantization directly but result from the exponential opening of mode coupling for modes with small quantum numbers at certain values of L, Eq. (1).

- The peaks on the curves $D(L)$ or $D(E)$ can help to extract the values of the size of thickness fluctuations R directly from the experimental data on diffusion. The curves $D(L)$ at constant R/Λ show how and when the standard QSE is replaced by the new QSE oscillations.

- The amplitudes of the QSE oscillations provide information on the contribution of individual modes to the quasiparticle diffusion and the role of mode coupling in scattering.

- Most importantly, the conclusions about the importance of mode coupling in scattering by large-scale inhomogeneities for QSE are fairly general and do not depend much on the origin of inhomogeneities or type of quasiparticles. For example, similar anomalous QSE should be observed when the scattering is caused by bulk inhomogeneities with a large correlation radius.

ACKNOWLEDGMENTS

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