

# Transport in $^3\text{He} - ^4\text{He}$ Mixtures in Restricted Geometry

A. E. Meyerovich

*Department of Physics, University of Rhode Island, Kingston, RI 02881*

## Abstract

The effect of wall scattering on transport in dilute degenerate  $^3\text{He} - ^4\text{He}$  mixtures in quasi-2D flow channels or films is discussed. The calculation of the quasiparticle mean free path combines particle-wall and particle-particle collisions including the interference between them. The spin polarization affects the wall-driven contribution by changing bulk mean free path and particle wavelength. The expressions for the wall contribution to transport coefficients are especially transparent in the limiting cases of large and small bulk mean free paths. The calculated temperature, concentration, and polarization dependences of the transport coefficients allow one to extract parameters of surface roughness from experimental data on transport.

## I. INTRODUCTION

Rapid progress in material science, vacuum and low temperature technologies, system miniaturization, *etc.* leads to proliferation of ballistic and semi-ballistic systems in which particle free paths become comparable to or even exceed the system size. It is difficult to overestimate the importance of wall scattering for transport processes in such systems. Usually, the wall scattering involves a convoluted combination of processes of different physical nature such as changes in energy spectra near the walls, stick-slip motion and particle accommodation, scattering by surface roughness and impurities, *etc.* Very rarely these physical phenomena can be unambiguously disentangled clearing the way to a full understanding of

the surface processes.

${}^3\text{He}$  is a convenient tool for the study of the effect of wall scattering on transport [1]. In contrast to many other liquids and solids, one can easily scan a wide range of particle mean free paths by simply changing the temperature. Low-temperature  ${}^3\text{He} - {}^4\text{He}$  mixtures provide an additional flexibility: the free paths and particle wavelengths can be easily adjusted by changing the  ${}^3\text{He}$  concentration. What is more, the formation of thin  ${}^4\text{He}$ -rich layers near the walls can prevent energy and magnetic accommodation of  ${}^3\text{He}$  quasiparticles on the walls. As a result, one gets an experimental access to a unique, almost model system with locally specular scattering of  ${}^3\text{He}$  quasiparticles with the simplest, practically quadratic energy spectrum.

Spin polarization of  ${}^3\text{He} - {}^4\text{He}$  mixtures adds even more experimental options. One of the most striking results of spin polarization is a dramatic increase in particle mean free paths and related transport coefficients [2]. By changing the spin polarization, one can scan the whole range of bulk mean free paths from the relatively small ones to the ones exceeding the channel width. As a result, one can prepare a system with an almost arbitrary relation between the bulk mean free path, wall-restricted free path, and system size. If the wall scattering is diffuse, then the scattering by the walls simply restricts the polarization-driven increase of the mean free path by the interwall spacing. In systems with locally specular scattering, such as  ${}^3\text{He} - {}^4\text{He}$  mixtures, the situation depends explicitly on the wall roughness and is much more complex. In such systems, the effective mean free path is determined by the interference between bulk and boundary scattering and can exceed considerably the distance between the walls. Since the particle wave lengths, particle-particle collisions, and effects of wall scattering are affected by spin polarization, the polarization dependence of the effective mean free path could be highly non-trivial.

This paper deals with the effect of the locally specular scattering by random rough walls on transport of  ${}^3\text{He}$  quasiparticles in  ${}^3\text{He} - {}^4\text{He}$  mixtures with and without spin polarization.

## II. TRANSPORT IN QUASICLASSICAL SYSTEMS WITH BULK AND WALL SCATTERING

Recently we reported a diagrammatic study of transport in quantized systems with both bulk and boundary scattering [3]. The important feature was a consistent description of interference between bulk and boundary scattering processes. In this section, we adapt these ultra-quantum results to quasiclassical systems and apply them to unpolarized dilute  ${}^3\text{He} - {}^4\text{He}$  mixtures.

We consider degenerate helium mixtures,  $T \ll T_F$ , where  $T_F = p_F^2/2m^* = (3\pi^2 N_3)^{2/3}/2m^*$  and  $p_F$  are the Fermi energy and momentum of  ${}^3\text{He}$  quasiparticles,  $N_3$  is the number of  ${}^3\text{He}$  particles per unit volume, and  $m^*$  is the effective mass of a single  ${}^3\text{He}$  quasiparticle (for typical values of parameters, see reviews [2,4]). We are interested in transport through a thin quasi-2D flow channel or film of the (average) thickness  $L$ . The random inhomogeneities of the walls are characterized by the correlation function  $\zeta(y, z)$  (or its Fourier image - the power density spectral function -  $\zeta(\mathbf{q})$ ; for precise definitions see Refs. [5,6]).

Though most of the calculations can often be performed for a surface correlator of a general type, in final results we assume that the correlations are Gaussian,

$$\zeta(\mathbf{s}) = \ell^2 \exp(-s^2/2R^2), \quad \zeta(\mathbf{q}) = 2\pi\ell^2 R^2 \left(-q^2 R^2/2\hbar^2\right) \quad (1)$$

where  $\ell$  and  $R$  play the role of the amplitude (height) and correlation radius (size) of surface inhomogeneities. In practice, the correlation function of surface inhomogeneities is not always Gaussian [7,8]. However, the exact structure of the surface correlator is less significant for dilute helium mixtures with longwave quasiparticles than for electron transport in metal or semiconductor films [9]. To reduce parameter clutter, we assume that the correlation function of random surface inhomogeneities is the same on both rough walls.

In Ref. [3] we calculated the effective relaxation time in ultrathin systems with quantum size effect (QSE). QSE is responsible for a split of the 3D spectrum  $\epsilon(\mathbf{p}) = p^2/2m^*$  into

a set of  $2D$  minibands,  $\epsilon_j(\mathbf{q}) = (1/2m^*) [(\pi j\hbar/L)^2 + q^2]$ . The diagrammatic equation for the Green's function of particles with bulk and boundary scattering reduces to the following expression for the effective relaxation time  $\tau_j^{(eff)}(\mathbf{q})$  for particles from each miniband  $\epsilon_j(\mathbf{q})$ :

$$\frac{1}{\tau_j^{(eff)}(\mathbf{q})} = \frac{1}{\tau_j^{(b)}(\mathbf{q})} + \sum_{j'=1}^S \int \frac{W_{jj'}(\mathbf{q}, \mathbf{q}') / \tau_{j'}^{(b)}(\mathbf{q}')}{(\epsilon_{j'}(\mathbf{q}') - T_F)^2 / \hbar^2 + (1/2\tau_{j'}^{(b)}(\mathbf{q}'))^2 (2\pi\hbar)^2} d\mathbf{q}', \quad (2)$$

where  $S$  is the total number of occupied or energetically accessible minibands  $\epsilon_j(\mathbf{q})$ , and  $\tau_j^{(b)}(\mathbf{q})$  is the bulk relaxation time in each miniband  $\epsilon_j$ . The wall-induced transition probability  $W_{jj'}(\mathbf{q}, \mathbf{q}')$  between the states  $\epsilon_j(\mathbf{q})$  and  $\epsilon_{j'}(\mathbf{q}')$  is determined by the correlation functions of surface inhomogeneities on both walls,  $\zeta_{11}$  and  $\zeta_{22}$ , and by the interwall correlation of surface inhomogeneities  $\zeta_{12}$ ,

$$W_{jj'}(\mathbf{q}, \mathbf{q}') = \frac{\pi^4 \hbar^2}{M^2 L^6} \left( \zeta_{11}(\mathbf{q} - \mathbf{q}') + \zeta_{22}(\mathbf{q} - \mathbf{q}') + 2(-1)^{j+j'} \zeta_{12}(\mathbf{q} - \mathbf{q}') \right) j^2 j'^2. \quad (3)$$

The Fermi momentum  $p_F$  of  ${}^3He$  quasiparticles in  ${}^3He - {}^4He$  mixtures is  $p_F/\hbar \sim 8.6 \cdot 10^9 X_3^{1/3} m^{-1}$  where  $X_3$  is the molar concentration of  ${}^3He$ . Thus, except for extremely low concentrations and/or nanochannels, the parameter  $p_FL/\hbar$  is usually large and transport is quasiclassical. [First signs of QSE in helium systems have been observed only recently in Ref. [10]]. The transformation of Eqs.(2), (3) to quasiclassical transport is fairly straightforward and requires to the replacement of summation over the miniband index  $j$  by the integration over the continuous variable  $p_x$ ,  $\pi j\hbar/L \rightarrow p_x$ . In addition, in the quasiclassical case  $p_FL/\hbar \gg 1$  one should disregard the interwall correlations  $\zeta_{12}$  [5]. Another important factor is that the *bulk* relaxation time  $\tau_b$  for  ${}^3He$  quasiparticles in dilute mixtures does not depend on particle momenta [2,4],

$$\tau_b = \frac{5\hbar^3}{4\pi m^*} \left( \frac{1}{aT} \right)^2 \quad (4)$$

where  $a \sim -0.9 \text{ \AA}$  is the  $s$ -wave scattering length of two  ${}^3He$  quasiparticles. The characteristic times for different transport processes can slightly differ from each other. We use the viscous bulk relaxation time  $\tau_b$  that determines the bulk viscosity in spin-polarized mixtures:

$$\eta_b = \tau_b N_3 p_F^2 / 5m^*. \quad (5)$$

Summarizing, Eqs.(2), (3) in quasiclassical  ${}^3\text{He} - {}^4\text{He}$  mixtures should be replaced by

$$\frac{1}{\tau^{(eff)}(\mathbf{p})} = \frac{1}{\tau_b} \left( 1 + \int \frac{W(\mathbf{p}, \mathbf{p}')}{(\epsilon(\mathbf{p}') - \mu)^2 / \hbar^2 + 1/4\tau_b^2} \frac{d\mathbf{p}'}{(2\pi\hbar)^3} \right), \quad (6)$$

with

$$W(\mathbf{p}, \mathbf{p}') = \frac{4\pi}{L} \left( \frac{\ell R}{\hbar M} \right)^2 p_x^2 p_x'^2 \exp\left(-(\mathbf{q} - \mathbf{q}')^2 R^2 / 2\hbar^2\right), \quad (7)$$

where we assume that the correlation function of surface inhomogeneities on both walls is Gaussian, Eq. (1). In the case of uniform distribution of particles over the angles, the effective times (6), (7) become

$$\frac{1}{\tau^{(eff)}} = \frac{1}{\tau_b} \left( 1 + \frac{16\ell^2 R^2 p_F^2}{\hbar^2 m^{*2} L} \int_0^\pi \cos^2 \theta d\theta \int \frac{p_x'^2 \exp\left(-(\mathbf{q} - \mathbf{q}')^2 R^2 / 2\hbar^2\right)}{(p^2 - p_F^2)^2 / m^{*2} \hbar^2 + 1/\tau_b^2} \frac{d\mathbf{p}'}{(2\pi\hbar)^3} \right), \quad (8)$$

where  $\theta$  is the angle between the momentum  $\mathbf{p}$  and  $x$  axis,  $\cos \theta = p_x / p_F$ .

Integration over the directions of  $\mathbf{q}'$  yields

$$\begin{aligned} \frac{1}{\tau^{(eff)}} &= \frac{1}{\tau_b} + \frac{1}{\tau_b} \frac{4\ell^2 R^2 p_F^2}{\pi^2 \hbar^5 m^{*2} L} \int_0^\pi \cos^2 \theta d\theta \int_{-\infty}^\infty p_x^2 dp_x' \\ &\times \int_0^\infty q' dq' \frac{\exp\left(- (q^2 + q'^2) R^2 / 2\hbar^2\right) I_0\left(qq' R^2 / \hbar^2\right)}{(p^2 - p_F^2)^2 / m^{*2} \hbar^2 + 1/\tau_b^2}, \end{aligned} \quad (9)$$

or, in dimensionless variables,

$$\frac{1}{\tau^{(eff)}} = \frac{1}{\tau_b} (1 + Q(t, u)), \quad Q = \frac{24\ell^2 R^2 N_3}{L} \int_0^\pi \cos^2 \theta e^{-(\sin^2 \theta + 1)u/2} \Psi(\theta) d\theta, \quad (10)$$

$$\Psi = \frac{1}{2} \int_0^\infty x^4 dx \int_0^\pi \cos^2 \phi \sin \phi d\phi \frac{e^{-(x^2 \sin^2 \phi - 1)u/2} I_0(ux \sin \theta \sin \phi)}{(x^2 - 1)^2 + 1/t^2} \quad (11)$$

$$u = p_F^2 R^2 / \hbar^2, \quad t^2 = \tau_b^2 p_F^4 / m^{*2} \hbar^2 = \frac{25}{\pi^2} \frac{(T_0/T)^4}{(3\pi^2 a^3 N_3)^{4/3}} \quad (12)$$

In these notations, the effective viscosity (5) reduces to

$$\eta = \frac{\hbar}{\pi} \frac{N_3}{(3\pi^2 N_3 a^3)^{2/3}} \left( \frac{T_F}{T} \right)^2 \frac{1}{1 + Q} = \frac{\eta_b}{1 + Q},$$

where  $\eta_b$  is the usual viscosity of mixtures is unrestricted geometry (5).

According to the definitions (12),  $u$  is the square of the ratio of the size of the surface inhomogeneities to the particle wavelength, and  $t$  is the dimensionless bulk relaxation time.

In other words,  $1/u$  characterizes the effectiveness of wall scattering and  $1/t$  - effectiveness of bulk collisions.

The integrand  $\Psi(\theta)$  in Eq. (11) can be rewritten as

$$\Psi = \int_0^\infty dy y^4 e^{-(y^2-1)u/2} I_0(uy \sin \theta) F(y, t), \quad (13)$$

$$F(y, t) = \int_0^{\pi/2} \frac{\cos^2 \phi d\phi}{(y^2 - \sin^2 \phi)^2 + \sin^4 \phi/t^2}. \quad (14)$$

The integral  $F(y, t)$  (14) can easily be evaluated:

$$F(y, t) = \frac{\sqrt{2}\pi}{4} \frac{1}{y^3 \left[ \sqrt{(y^2 - 1)^2 + 1/t^2} + (y^2 - 1) \right]^{1/2}} \quad (15)$$

making

$$\Psi(\theta, t, u) = \frac{\sqrt{2}\pi}{4} \int_0^\infty \frac{y e^{-(y^2-1)u/2} I_0(uy \sin \theta) dy}{\left[ \sqrt{(y^2 - 1)^2 + 1/t^2} + (y^2 - 1) \right]^{1/2}}. \quad (16)$$

The behavior of the function  $\Psi(\theta, t, u)$  (16) is determined by the ratio of parameters  $u$  and  $t$ . The denominator in this integral creates a peak of the width  $1/t$ , while the numerator is a peak of the width  $1/u$ . Surprisingly, the ratio of these two parameters does not involve the channel width  $L$  and contains only the correlation radius of inhomogeneities. If  $t \gg u$ , *i.e.*, at low temperatures

$$(T_F/T)^2 \gg (R/a)^2 X^{4/3} \quad (17)$$

( $X$  is the molar concentration of  ${}^3\text{He}$  in the mixture), the denominator in Eq.(11) is a much sharper function. Since the Fermi temperature in mixtures is proportional to  $X^{2/3}$ , the  ${}^3\text{He}$  concentration disappears from this condition which is equivalent at SVP to

$$(T/2.64)^2 \ll (a/R)^2 \quad (18)$$

where  $T$  is in  $K$ . In this case,

$$\Psi(\theta, t \gg u) = \frac{\pi}{2} t \int_0^1 e^{-(y^2-1)u/2} I_0(uy \sin \theta) \sqrt{1 - y^2} y dy. \quad (19)$$

This integral can easily be evaluated analytically at  $u \ll 1$  :

$$\Psi(t, 1 \gg u) = \frac{\pi}{2} t \int_0^1 \sqrt{1-y^2} dy = \frac{\pi}{6} t \quad (20)$$

( $I_0(0) = 1$ ). Then the wall-driven correction to the relaxation time is

$$Q(t, 1 \gg u) = \pi^2 \frac{\ell^2 R^2 N_3}{L} t, \quad (21)$$

and the effective relaxation time becomes

$$\frac{1}{\tau^{(eff)}} = \frac{1}{\tau_b} + 4\pi^2 \frac{\ell^2 R^2 N_3}{L} \frac{T_F}{\hbar} \quad (22)$$

In this case, the bulk- and wall-driven relaxation times are completely decoupled from each other, and the effective time is given by the Matthiessen's rule,  $1/\tau^{(eff)} = 1/\tau_b + 1/\tau_w$ .

This pure wall-induced contribution to the relaxation time,

$$\frac{1}{\tau_w} = 4\pi^2 \frac{\ell^2 R^2 N_3}{L} \frac{T_F}{\hbar} \quad (23)$$

is temperature independent and is proportional to the concentration  ${}^3He$  in the mixture to the power 5/3. Numerically, at SVP

$$\frac{1}{\tau_w} = 3.16 \cdot 10^7 \frac{\ell^2 R^2}{L} X_3^{5/3} s^{-1} \quad (24)$$

with  $\ell, R$ , and  $L$  in  $nm$ . For comparison,

$$\frac{1}{\tau_b} = 3.07 \frac{T^2 m^* a^2}{\hbar^3} = 5.35 \cdot 10^{10} T^2 s^{-1} \quad (25)$$

with  $T$  in  $K$ . If, for example,  $X \sim 10^{-3}$ ,  $T \sim 1 mK$ , the wall-driven relaxation time is shorter than the bulk relaxation time at

$$\frac{\ell^2 R^2}{L} > 100$$

where, again,  $\ell, R$ , and  $L$  are measured in  $nm$ . This condition is reasonable.

Since  $u = p_F^2 R^2 / \hbar^2$ , the condition  $u \ll 1$  corresponds to

$$u = (3\pi^2 N_3)^{2/3} R^2 = 75 R^2 X^{2/3} \ll 1$$

( $R$  in  $nm$ ). This condition seems realistic for walls with nanoscale inhomogeneities. Outside of the region  $u \ll 1$ , but still at  $u \ll t$  (18), the wall contribution to relaxation (10) is

$$Q(t \gg u) = \pi^2 \frac{\ell^2 R^2 N_3}{L} t f(u), \quad (26)$$

$$\frac{1}{\tau_w} = 4\pi^2 \frac{\ell^2 R^2 N_3 T_F}{L \hbar} f(u), \quad (27)$$

$$f(u) = \frac{12}{\pi} \int_0^\pi \cos^2 \theta e^{-\sin^2 \theta u/2} \int_0^1 e^{-y^2 u/2} I_0(uy \sin \theta) \sqrt{1-y^2} y dy d\theta. \quad (28)$$

Function  $f(u)$  is a featureless monotonic function. The plot of this function is given in Figure 1.

In the opposite case of not very low temperatures (or large-scale inhomogeneities)  $t \ll u$ ,

$$(T_F/T)^2 \ll (R/a)^2 X^{4/3}, \quad (29)$$

*i.e.*, when

$$(T/2.64)^2 \gg (a/R)^2 \quad (30)$$

the denominator in Eq.(16) is almost a constant with  $y \approx 1$  and

$$\Psi(\theta, t \ll u) = \frac{\sqrt{2}\pi}{4u} t^{1/2} e^{(1+\sin^2 \theta)u/2}, \quad (31)$$

$$Q(t \ll u) = \frac{\pi^2}{2\sqrt{2}} \frac{12\ell^2 R^2 N_3 t^{1/2}}{L u} \quad (32)$$

In this case,

$$\frac{1}{\tau^{(eff)}} = \frac{1}{\tau_b} + 6\sqrt{\frac{2}{5}} \pi^{5/2} \frac{|a| \ell^2 N_3 \hbar T}{L p_F \hbar}. \quad (33)$$

Numerically, if  $|a| = 0.09 nm$ , this equation is equivalent to

$$\frac{1}{\tau^{(eff)}} = \frac{1}{\tau_b} + 4.5 \cdot 10^6 \frac{\ell^2}{L} X^{2/3} T s^{-1} \quad (34)$$

with  $T$  in  $K$ ,  $\ell$  and  $L$  in  $nm$ . The numerical estimate for  $1/\tau_b$  is given by Eq.(25).

The temperature and concentration dependences of the wall contribution to the effective relaxation time for these two limiting cases, (22) and (33), are so simple that plotting of these functions is unnecessary. The difference between them is also absolutely clear.

An interesting feature is the dependence of the wall contribution on the bulk relaxation time: while the wall term in Eq.(22) does not depend on bulk relaxation parameters, the wall term in Eq.(33) is inversely proportional to the square root of the bulk mean free path. The explanation is simple. In the former case, the bulk mean free path is larger than the wall-driven contribution and the bulk and wall scattering processes are independent from each other. In opposite case (29), (30), the bulk mean free path is small and the particle returns to the wall after one or more bulk scattering acts rather than after the reflection from the opposite wall. This is the source of a large *classical* interference between bulk and wall scattering processes that is responsible for the peculiar dependence of the wall contribution on the bulk scattering parameters.

### III. TRANSPORT IN SPIN-POLARIZED MIXTURES

Spin polarization of  ${}^3\text{He} - {}^4\text{He}$  mixtures results in change in particle wavelengths and, more important, in drastic changes in bulk mean free paths [2]. In addition, in spin-polarized mixtures spin-up and spin-down quasiparticles have different dynamic and kinetic properties. The polarization dependence of the *bulk* viscous relaxation times of the spin-up and spin-down particles is given by the equation

$$\tau_{\pm} = \frac{\hbar^3}{2\pi m^*} \left( \frac{1}{aT} \right)^2 \frac{5}{5 - 3(c_{\mp}/c_{\pm})^{2/3}} \frac{c_{\pm}}{c_{\mp}}, \quad (35)$$

where  $c_{\pm}$  are the relative "concentrations" of spin-up and spin-down particles,  $c_{\pm} = N_{\pm}/N_3$ . These concentrations and the overall degree of the spin polarization  $\Delta$  of dilute degenerate mixtures in external magnetic field  $H$  are determined by the equations

$$\begin{aligned} c_+^{2/3} - c_-^{2/3} &= 2^{-2/3} h_0 + \frac{8\pi}{(6\pi^2)^{2/3}} a N_3^{1/3} (c_+ - c_-), \\ \Delta &= c_+ - c_-, \quad c_+ + c_- = 1, \quad h_0 = 2\beta H/T_F, \end{aligned} \quad (36)$$

where  $\beta = 0.08 \text{ mK}/T$  is the nuclear magnetic moment of  ${}^3\text{He}$ , and  $a \sim -0.09 \text{ nm}$  is the *s*-wave scattering length for  ${}^3\text{He}$  quasiparticles. The *bulk* relaxation times  $\tau_{\pm}$  (35) determine bulk viscosity in spin-polarized mixtures:

$$\eta = \eta_+ + \eta_-, \quad \eta_{\pm} = \tau_{\pm} N_{\pm} p_{\pm}^2 / 5m^*, \quad (37)$$

where  $p_{\pm} = (2c_{\pm})^{1/3} p_F$  are the Fermi momenta for spin-up and spin-down components of the mixture.

If the walls are non-magnetic, the scattering by the walls does not affect the value of spin and the presence of two different spin species leads simply to the doubling of equations. Then the wall-induced change in the relaxation times for both species is given by the equation similar to Eq.(6):

$$\frac{1}{\tau_{\pm}^{(eff)}(\mathbf{p})} = \frac{1}{\tau_{\pm}} \left( 1 + \int \frac{W(\mathbf{p}, \mathbf{p}')}{(\epsilon_{\pm}(\mathbf{p}') - \mu)^2 / \hbar^2 + 1/4\tau_{\pm}^2 (2\pi\hbar)^3} d\mathbf{p}' \right), \quad (38)$$

with  $W(\mathbf{p}, \mathbf{p}')$  from Eq.(7). Here we took into account that in dilute polarized mixtures both *bulk* relaxation times  $\tau_{\pm}$  for spin-up and spin-down quasiparticles (35) do not depend on particle momenta.

The integral (38) is almost the same as (6) – (11):

$$\begin{aligned} \frac{1}{\tau_{\pm}^{(eff)}} &= \frac{1}{\tau_{\pm}} (1 + Q_{\pm}), \quad Q_{\pm} = \frac{24\ell^2 R^2 N_3}{L} 2c_{\pm} \int_0^{\pi} \cos^2 \theta e^{-(\sin^2 \theta + 1)u/2} \Psi_{\pm}(\theta) d\theta, \\ \Psi_{\pm}(\theta) &= \frac{\sqrt{2}\pi}{4} \int_0^{\infty} \frac{y e^{-(y^2-1)u_{\pm}/2} I_0(u_{\pm} y \sin \theta) dy}{\left[ \sqrt{(y^2-1)^2 + 1/t_{\pm}^2} + (y^2-1) \right]^{1/2}}, \end{aligned} \quad (39)$$

$$u_{\pm} = (2c_{\pm})^{2/3} p_F^2 R^2 / \hbar^2 = u (2c_{\pm})^{2/3}, \quad t_{\pm} = \tau_{\pm} p_{\pm}^2 / M\hbar = t \frac{2(2c_{\pm})^{2/3} c_{\mp}}{5 - 3(c_{\mp}/c_{\pm})^{2/3} c_{\mp}} \quad (40)$$

Finally, the effective viscosity reduces to

$$\begin{aligned} \eta &= 2^{2/3} (N_3 p_F^2 / 5m^*) \left[ c_+^{5/3} \frac{\tau_+}{1 + Q_+} + c_-^{5/3} \frac{\tau_-}{1 + Q_-} \right] \\ &= \eta_b \left[ \frac{(2c_+)^{5/3} c_+}{5 - 3(c_-/c_+)^{2/3} c_-} \frac{1}{1 + Q_+} + \frac{(2c_-)^{5/3}}{2(1 + Q_-)} \right], \end{aligned} \quad (41)$$

where  $\eta_b$  is again the bulk viscosity of unpolarized mixtures.

The behavior of the integral  $\Psi_{\pm}$  (39) is determined by the relation between of parameters  $u_{\pm}$  and  $t_{\pm}$ . If  $t_{\pm} \gg u_{\pm}$ , *i.e.*, at low temperatures and/or very high polarizations

$$(T_F/T)^2 \gg (R/a)^2 X^{4/3} c_{\mp}, \quad (42)$$

(cf. Eqs.(17), (18)), the wall-driven corrections to the relaxation time  $Q_{\pm}$  are

$$Q_{\pm}(t \gg u) = \pi^2 \frac{\ell^2 R^2 N_3}{L} t_{\pm} f(u_{\pm}), \quad (43)$$

where the function  $f(u)$  is defined by Eq. (28) and is displayed in Figure 1. Numerically, at SVP  $u_{\pm} = 75R^2 X^{2/3} (2c_{\pm})^{2/3}$  (with  $R$  in  $nm$ ).

If  $u_{\pm} \ll 1$ ,

$$\Psi(t_{\pm}, 1 \gg u_{\pm}) = \frac{\pi}{2} t_{\pm} \int_0^1 \sqrt{1-y^2} y dy = \frac{\pi}{6} t_{\pm} \quad (44)$$

and

$$Q_{\pm}(t_{\pm}, 1 \gg u_{\pm}) = 4\pi^2 \frac{\ell^2 R^2 N_3}{L} c_{\pm}^2 t_{\pm} = 4\pi^2 \frac{\ell^2 R^2 N_3}{L} t_{\pm} \frac{2(2c_{\pm})^{8/3}}{5-3(c_{\mp}/c_{\pm})^{2/3}} \frac{c_{\pm}}{c_{\mp}} \quad (45)$$

Then the relaxation times

$$\frac{1}{\tau_{\pm}^{(eff)}} = \frac{1}{\tau_{\pm}} + 4\pi^2 (2c_{\pm})^{8/3} \frac{\ell^2 R^2 N_3 T_F}{L \hbar} \quad (46)$$

$$\approx \frac{1}{\tau_{\pm}} + 3.16 \cdot 10^7 \frac{\ell^2 R^2}{L} (2c_{\pm})^{8/3} X_3^{5/3} s^{-1} \quad (47)$$

with  $\ell, R$ , and  $L$  in  $nm$ .

In this case, the bulk- and wall-driven relaxation times are independent, and the effective time is given by the Matthiessen's rule. The polarization dependence of the wall contribution is given by a trivial factor  $(2c_{\pm})^{8/3}$ .

In the opposite case of not very low temperatures (or large-scale inhomogeneities) and moderate polarizations  $t_{\pm} \ll u_{\pm}$ ,

$$(T_F/T)^2 \ll (R/a)^2 X^{4/3} c_{\mp}, \quad (48)$$

(cf. Eqs.(29), (30)), the integrals are equal to

$$\begin{aligned} \Psi_{\pm}(\theta, t_{\pm} \ll u_{\pm}) &= \frac{\sqrt{2\pi}}{4u_{\pm}} t_{\pm}^{1/2} e^{(1+\sin^2 \theta)u/2}, \\ Q_{\pm} &= \frac{\pi^2}{\sqrt{2}} \frac{12\ell^2 R^2 N_3}{L} \frac{c_{\pm} t_{\pm}^{1/2}}{u_{\pm}} \end{aligned} \quad (49)$$

In this case

$$\frac{1}{\tau_{\pm}^{(eff)}} = \frac{1}{\tau_{\pm}} + 6\pi^{5/2} q_{\pm} \frac{a\ell^2 N_3 \hbar T}{L p_F \hbar} \quad (50)$$

$$\simeq \frac{1}{\tau_{\pm}} + 4.5 \cdot 10^6 \frac{\ell^2}{L} X^{2/3} T q_{\pm} \text{ s}^{-1}, \quad (51)$$

$$q_{\pm} = (2c_{\pm})^2 \left( \frac{5}{2} - \frac{3}{2} \left( \frac{c_{\mp}}{c_{+}} \right)^{2/3} \right)^{1/2} \left( \frac{c_{\mp}}{c_{+}} \right)^{1/2}. \quad (52)$$

with  $T$  in  $K$ ,  $\ell$  and  $L$  in  $nm$ . All polarization dependence is contained in the factors  $q_{\pm}$  (52). Functions  $q_{\pm}(\Delta)$  are plotted in Figure 2.

Interestingly, the wall-induced contributions to the inverse relaxation time  $1/\tau_{\pm}^{(eff)}$  (50) formally goes to zero when the mixture becomes completely polarized,  $q_{\pm}(\Delta \rightarrow 1) \rightarrow 0$ . Of course, in reality these wall-induced contributions do not disappear, at least for spin-ups, because at high enough polarization one always leaves the hydrodynamic regime (48) and gets into the opposite, ballistic regime (42),(46). Note, that  $q_{-}$  goes to zero at  $c_{-} \rightarrow 0$  much faster than  $q_{+}$ .

At high spin polarization one can observe a peculiar "mixed" low-temperature regime between the limiting cases (42) and (48) when  $c_{-}$  is so small that the majority spins (spin-ups) are in the ballistic regime (42) with the relaxation time given by Eq.(46) while the spin-downs are in the hydrodynamic regime (48) and have a relatively small bulk mean free path with the effective time  $\tau_{-}^{(eff)}$  given by Eq.(50).

#### IV. SUMMARY

In summary, we report the wall contribution to low-temperature viscosity of superfluid  ${}^3He - {}^4He$  mixtures in narrow quasi-2D flow channels. Due to the peculiar properties of the superfluid mixtures, a potential experimental study provides one with a unique opportunity of measuring the effect of surface roughness on transport phenomena. The low-temperature scattering of  ${}^3He$  quasiparticles from the walls is almost completely specular locally and is a sensitive probe of the correlation function of surface inhomogeneities. Temperature,

concentration, and spin polarization dependences of the transport coefficients allow one to monitor the system in a wide range of bulk mean free paths and particle wavelengths. The equations for these dependences are very simple and could allow one to extract parameters of wall roughness directly from the transport data.

The calculations are performed in a wide range of bulk free paths and particle wavelengths though without quantum size effect (systematic transport measurements in QSE conditions are not yet possible). The results are especially simple in the limiting cases of ballistic and hydrodynamic regimes, (17) and (29), when the bulk collisions are either less or more effective than the wall scattering. In these two cases, the concentration and temperature dependences of the wall-driven terms (24) and (34) are quite transparent and readily distinguishable. Interestingly, the conditions (17), (29) that separate the ballistic and hydrodynamic limits are determined solely by the temperature and the correlation radius of surface inhomogeneities; the  $^3\text{He}$  concentration and the width of the flow channel do not enter into these conditions. Numerical estimates show that both regimes seem to be accessible experimentally.

Spin polarization of the  $^3\text{He}$  quasiparticle system adds another tool for experimental study of the wall contribution to transport. Apart from a distinct polarization dependence of the viscosity, high polarization can result in a peculiar regime in which the spin-up quasiparticles are ballistic while the spin-downs remain hydrodynamic. In this regime, the wall contributions from spin components of the quasiparticle system are distinctly different from each other.

In general, the difference in the polarization dependence between Eqs.(46) and (50) is quite striking and can provide an easier mechanism for the identification of the situation and for finding the parameters of surface inhomogeneities from the experimental data than the study of the concentration and temperature dependence of the viscosity in unpolarized mixtures (24) and (34).

The results of the paper can be applied to pure normal liquid  $^3\text{He}$  with some caution. The main constraint is the requirement of quasielastic scattering of  $^3\text{He}$  by the walls. Depending

on the wall material and preparation, the wall scattering for pure  ${}^3\text{He}$  often exhibits energy or magnetic accommodation and leads to the stick-slip motion. If this is the case, the results of this paper cannot be used. Otherwise, in the cases of walls with quasielastic scattering, the only expressions that should be avoided are the ones that rely explicitly on the low density approximation for the relaxation time. Experimentally, the quasielastic scattering can be ensured by coating the walls by hydrogen or  ${}^4\text{He}$ .

The results can also be modified for quasiclassical transport in metal films with near spherical Fermi surfaces. For such films, one can use the equations of Section II though without relying on Eq.(6) for the bulk relaxation time  $\tau_b$ .

## V. ACKNOWLEDGMENTS

I am grateful to Reyer Jochemsen for numerous helpful discussions of the experimental opportunities. The work is supported by NSF grants INT-9724728 and DMR-0077266.

## REFERENCES

- [1] H. Smith, *Long Mean Free Paths in Quantum Fluids*, in: Progr.Low.Temp.Phys., ed. D.F.Brewer, **11**, 75-126 (1987)
- [2] A. E. Meyerovich, *Spin-Polarized Phases of  $^3\text{He}$* , in: Helium Three, eds. W.P.Halperin, L.P.Pitaevskii (North Holland, Amsterdam, 1991), p.p.757-879; *Spin-Polarized Phases of  $^3\text{He} - ^4\text{He}$  Solutions*, in: Progr.Low.Temp.Phys., ed. D.F.Brewer, **11**, 1 - 75 (1987)
- [3] A. E. Meyerovich and A. Stepaniants, J.Phys.: Cond. Matter **12**, 5575 (2000)
- [4] J. Owers-Bradley, Rep. Prog. Phys., **60**,1173 (1997)
- [5] A. E. Meyerovich, and A. Stepaniants, Phys.Rev. B **58**, 13 243 (1998)
- [6] A. E. Meyerovich, , and A. Stepaniants, A., Phys.Rev. B **60**, 9129 (1999)
- [7] J.A.Ogilvy, *Theory of Wave Scattering from Random Surfaces* (Adam Hilger, Bristol) 1991
- [8] R.M.Feenstra *et al*, Phys.Rev.Lett. **72**, 2749 (1994)
- [9] A. E. Meyerovich, I. Ponomarev, to be published
- [10] S.L. Phillipson, A.M. Guenault, S.N. Fisher, G.R.Pickett, and P.J.Y. Thibault, Nature **395**, 578 (1998); P. A. Reeves, A. M. Guenault, S. N. Fisher, G. R. Pickett, and G. Tvalashvili, Physica B **284-288** 319 (2000)

## FIGURE CAPTIONS

Figure 1. Function  $f(u)$ , Eq. (28).

Figure 2. Polarization dependence of the factors  $q_+(\Delta)$  and  $q_-(\Delta)$ , Eq. (52).