Electric Potential Energy of Two Point Charges

Consider two different perspectives:

#1a Electric potential when \( q_1 \) is placed: \( V(\mathbf{r}_2) \doteq V_2 = k \frac{q_1}{r_{12}} \)

Electric potential energy when \( q_2 \) is placed into potential \( V_2 \): \( U = q_2 V_2 = k \frac{q_1 q_2}{r_{12}} \)

#1b Electric potential when \( q_2 \) is placed: \( V(\mathbf{r}_1) \doteq V_1 = k \frac{q_2}{r_{12}} \)

Electric potential energy when \( q_1 \) is placed into potential \( V_1 \): \( U = q_1 V_1 = k \frac{q_1 q_2}{r_{12}} \).

#2 Electric potential energy of \( q_1 \) and \( q_2 \):

\[
U = \frac{1}{2} \sum_{i=1}^{2} q_i V_i,
\]

where \( V_1 = k \frac{q_2}{r_{12}} \), \( V_2 = k \frac{q_1}{r_{12}} \).
# Electric Potential Energy of Three Point Charges

#1 Place $q_1$, then $q_2$, then $q_3$, and add all changes in potential energy:

$$U = 0 + k \frac{q_1 q_2}{r_{12}} + k \left( \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) = k \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right).$$

#2 Symmetric expression of potential energy $U$ in terms of the potentials $V_i$ experienced by point charges $q_1$:

$$U = \frac{1}{2} \sum_{i=1}^{3} q_i V_i = k \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right),$$

where

$$V_1 = k \left( \frac{q_2}{r_{12}} + \frac{q_3}{r_{13}} \right),$$

$$V_2 = k \left( \frac{q_1}{r_{12}} + \frac{q_3}{r_{23}} \right),$$

$$V_3 = k \left( \frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right).$$