Flexibility is an intrinsic property of polymers. Consider an ideal polymer chain with $N$ links of length $a$. Its contour length is $L = Na$. If we divide that chain into segments of length $l \geq a$ then, with growing size of these segments, the joints become effectively less constrained and less stiff. At the Kuhn segment length $l_K$ the joints become effectively free. The mean-square distance of a freely-jointed chain (FJC) is $\langle R^2 \rangle = N a^2 = La$ [pln50]. The natural definition of the Kuhn segment length, therefore, is [pln51]

$$l_K = \frac{\langle R^2 \rangle}{L}.$$ 

The Kuhn segment length $l_K$ is a measure for the stiffness of the polymer chain just as the persistence length $l_p$ investigated in [pex28] is. However, the two measures are not identical. The Kuhn segment length is easier to determine experimentally and theoretically but the persistence length has a more direct physical meaning. Here we explore the functional relation between $l_K$ and $l_p$ for an ideal polymer chain with persistent flexibility. On a mesoscopic scale we describe the conformation of the polymer by a vector function $\vec{r}(s)$ and replace the local bond vector $\vec{a}_i$ by the vector function $\vec{u}(s) = d\vec{r}/ds$ with $s$ as defined in [pex28]. The end-to-end distance vector and its mean-square value can thus be expressed as follows:

$$\vec{R} = \int_0^L ds \vec{u}(s), \quad \langle R^2 \rangle = \int_0^L ds \int_0^L ds' \langle \vec{u}(s) \cdot \vec{u}(s') \rangle.$$ 

To calculate the latter we infer from [pex28] the relation

$$\langle \vec{u}(s) \cdot \vec{u}(s') \rangle = \langle \cos \theta(s - s') \rangle = e^{-|s-s'|/\tilde{l}}.$$ 

Perform the double integral to obtain an analytic expression of the scaled Kuhn segment length $l_K/L$ as a function of the scaled persistence length $l_p/L$. Show in particular that for very long polymers ($L \gg \tilde{l}$), we have $l_K \simeq 2l_p$ and for very short polymers ($L \ll l_p$) we have $l_K \simeq L$. Plot $l_K/L$ versus $l_p/L$ over the range $0 < l_p/L < 3$ to illustrate this behavior.

[adapted from Grosberg and Khokhlov 1994]

Solution: