A battery providing an emf $\mathcal{E}$ with internal resistance $r$ is connected to an external resistor of resistance $R$ as shown.

For what value of $R$ does the battery deliver the maximum power to the external resistor?

- Electric current: $\mathcal{E} - Ir - IR = 0 \Rightarrow I = \frac{\mathcal{E}}{R + r}$
- Power delivered to external resistor: $P = I^2R = \frac{\mathcal{E}^2R}{(R + r)^2} = \frac{\mathcal{E}^2}{r} \frac{R/r}{(R/r + 1)^2}$
- Condition for maximum power: $\frac{dP}{dR} = 0 \Rightarrow R = r$