Electric Flux: Definition

Consider a surface $S$ of arbitrary shape in the presence of an electric field $\vec{E}$.

Prescription for the calculation of the electric flux through $S$:

- Divide $S$ into small tiles of area $\Delta A_i$.
- Introduce vector $\Delta \vec{A}_i = \hat{n}_i \Delta A_i$ perpendicular to tile.
  - If $S$ is open choose consistently one of two possible directions for $\Delta \vec{A}_i$.
  - If $S$ is closed choose $\Delta \vec{A}_i$ to be directed outward.
- Electric field at position of tile $i$: $\vec{E}_i$.
- Electric flux through tile $i$:
  \[ \Delta \Phi_i^{(E)} = \vec{E}_i \cdot \Delta \vec{A}_i = E_i \Delta A_i \cos \theta_i. \]
- Electric flux through $S$: $\Phi_E = \sum_i \vec{E}_i \cdot \Delta \vec{A}_i$.
- Limit of infinitesimal tiles: $\Phi_E = \int \vec{E} \cdot d\vec{A}$.
- Electric flux is a scalar.
- The SI unit of electric flux is Nm$^2$/C.